SIMULTANEOUS DIFFERENTIAL EQUATIONS

The differential Equation of the form
$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$
, where P, Q, R are functions of x, y, z is called Simultaneous equations.

W.R. **1st Method**: Let us choose some multipliers l, m, n such that
$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} = \frac{l\,dx + m\,dy + n\,dz}{lP + mQ + nR}$$
if $lP + mQ + nR = 0$ and hence $l\,dx + m\,dy + n\,dz = 0$

If it is an exact, then integrate and get general solution.

2nd Method: Take any two members (fractions) of the given equation and solve them.

Geometrical interpretation of $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$:

Geometrically the above equations represent a system of curves in space such that the direction ratios of the tangent to it at any point (x, y, z) are proportional to P, Q, R.

Problem (24) Solve the simultaneous equations
$$\frac{l\,dx}{mn(y-z)} = \frac{m\,dy}{nl(z-x)} = \frac{n\,dz}{lm(x-y)}$$

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Ans: Taking l, m, n as multipliers, we have
$$\frac{l\,dx}{mn(y-z)} = \frac{m\,dy}{nl(z-x)} = \frac{n\,dz}{lm(x-y)} = \frac{l^2dx + m^2dy + n^2dz}{lmn(y-z+z-x+x-y)}$$

i.e $\frac{l\,dx}{mn(y-z)} = \frac{m\,dy}{ln(z-x)} = \frac{n\,dz}{lm(x-y)} = \frac{l^2dx+m^2dy+n^2dz}{0}$

$\Rightarrow l^2dx + m^2dy + n^2dz = 0$, Integrating

$\Rightarrow l^2x + m^2y + n^2z = a$, where a is an arbitrary constant ——— (1)

Again taking multipliers lx, my, nz, then we have

$\frac{l\,dx}{mn(y-z)} = \frac{m\,dy}{nl(z-x)} = \frac{n\,dz}{lm(x-y)} = \frac{l^2x\,dx + m^2y\,dy + n^2z\,dz}{lmn\{x(y-z)+y(z-x)+z(x-y)\}}$

$\Rightarrow$ Each ratio $= \frac{l^2x\,dx+m^2y\,dy+n^2z\,dz}{lmn(xy-xz+yz-xy+xz-yz)}$

$\Rightarrow$ Each ratio $= \frac{l^2x\,dx+m^2y\,dy+n^2z\,dz}{0}$

$\Rightarrow l^2x\,dx + m^2y\,dy + n^2z\,dz = 0$, integrating

$l^2\frac{x^2}{2} + m^2\frac{y^2}{2} + n^2\frac{z^2}{2} = \frac{b}{2}$

$\Rightarrow l^2x^2 + m^2y^2 + n^2z^2 = b$, where b is any constant ——— (2)

(1) and (2) together give the complete solution.

Problem (25) Solve $\frac{dx}{y} = \frac{dy}{x} = \frac{dz}{xyz^2(x^2-y^2)}$

Ans Taking Ist and second ratios,

$x\,dx = y\,dy \Rightarrow \int x\,dx = \int y\,dy$

$\Rightarrow \frac{x^2}{2} = \frac{y^2}{2} + \frac{c}{2} \Rightarrow x^2 - y^2 = c$ ——— (1)

Again taking Ist and 3rd ratioes, we have

$\frac{dx}{y} = \frac{dz}{xyz^2(x^2-y^2)} \Rightarrow \frac{dx}{y} = \frac{dz}{xyz^2 \cdot c}$ by (1)

$\Rightarrow cx\,dx = z^{-2}dz \Rightarrow c\int x\,dx = \int z^{-2}dz \Rightarrow \frac{cx^2}{2} = \frac{z^{-2+1}}{-2+1} + a$

$\Rightarrow (x^2-y^2)\frac{x^2}{2} = -\frac{1}{z} + a$ ——— (2). (1) & (2) gives the complete solution

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