Numericals based on Carnot's Engine

Q. I A carnot engine operates with an efficiency of 50%. If during each cycle it rejects 150 cal to a reservoir of heat at 30°C, then (i) what is the temp of other reservoir? (ii) How ~~does~~ much work does it carry out per cycle?

Solⁿ:- According to question

$Q_2 = 150$ cal, $T_2 = 30 + 273 = 303$ K

$\eta = 50\% = 0.5$

Now

(i) ∵ $\eta = 1 - \frac{T_2}{T_1}$

$0.5 = 1 - \frac{303}{T_1}$ or $T_1 = \frac{303}{0.5}$

$\Rightarrow T_1 = 606$ K $= 606 - 273 = 333$°C

(ii) Also $\eta = 1 - \frac{Q_2}{Q_1}$

or $0.5 = 1 - \frac{150}{Q_1}$

$Q_1 = \frac{150}{0.5} = 300$ cal

∴ $W = Q_1 - Q_2$

$= 300 - 150$

$= 150$ cal $= 150 \times 4.2 = 630$ J

Ans

Q(2) A reversible engine converts one sixth of heat input into work. When the temp of the sink is reduced by 62°C, its efficiency is doubled. Find the temperature of the source and sink

Soln According to question

$$W = \frac{Q_1}{6}$$

$$\therefore \quad \eta = \frac{W}{Q_1} = 1 - \frac{T_2}{T_1}$$

or $\frac{1}{6} = 1 - \frac{T_2}{T_1}$

or $\frac{T_2}{T_1} = \frac{5}{6}$

$T_1 = \frac{6}{5} T_2 \quad \text{—(i)}$

Also if $T_2' = T_2 - 62$, $\eta' = \frac{1}{6} \times 2 = \frac{1}{3}$

Now $\eta' = 1 - \frac{T_2'}{T_1}$ $\quad T_1 = \text{const}$

$\frac{1}{3} = 1 - \frac{T_2 - 62}{T_1}$

or $\frac{T_2 - 62}{T_1} = \frac{2}{3}$ or $T_1 = \frac{3}{2}(T_2 - 62)$ —(ii)

From (i) & (ii) $1.5(T_2 - 62) = 1.2 T_2$

$0.3 T_2 = 62 \times 1.5$

$T_2 = \frac{62 \times 1.5}{0.3} = 310\ K$ Ans

From (i) $T_1 = 1.2 \times 310 = 372\ K$ Ans

Q.3 A Carnot engine, whose low temperature reservoir is at 7°C has an efficiency of 40%. It is desired to increase the efficiency by 50%. By how much degree must the temp of source be increased?

Sol^n: A/q $\eta = 40\% = 0.4$ here $T_2 = 7 + 273 = 280K$

$\because \eta = 1 - \frac{T_2}{T_1}$ or $0.4 = 1 - \frac{280}{T_1}$

or $T_1 = 466.67\ K$

when $\eta = 50\%$

$T_2 = 280\ K$

$T_1 = T_1'$ (Let)

Now $0.5 = 1 - \frac{280}{T_1'}$

or $T_1' = 560\ K$

Therefore change in temp of source, i.e

$\Delta T = T_1' - T_1$

$= 560 - 466.67$

$\boxed{\Delta T = 93.93\ K}$ Ans