(2)

UG - Sem VI Sastasha 14/5/2020

Tensor Calculus

7. Determinant: Let us consider a determinant

$$\begin{vmatrix} a_1^1 & a_2^1 & a_3^1 & a_4^1 \\ a_1^2 & a_2^2 & a_3^2 & a_4^2 \\ a_1^3 & a_2^3 & a_3^3 & a_4^3 \\ a_1^4 & a_2^4 & a_3^4 & a_4^4 \end{vmatrix} = a \text{ (say)}$$

Here a_ν^μ may be taken as the general element of this determinant. This determinant is also denoted by $|a_\nu^\mu|$. The suffixes μ and ν denote the row and the column respectively to which the element a_ν^μ belongs. The cofactor of the element a_ν^μ is denoted by A_μ^ν.

8. Transformation of Co-ordinates: We consider a transformation from one coordinate system (x^1, x^2, x^3, x^4) to another coordinate system (x'^1, x'^2, x'^3, x'^4), where $x'^i = x'^i(x^1, x^2, x^3, x^4)$, $i = 1,2,3,4$. The four functions x'^i of coordinates x^i are single valued continuous differentiable. The differentials (dx^1, dx^2, dx^3, dx^4) are transformed according to the equations

$$dx'^1 = \frac{\partial x'^1}{\partial x^1} dx^1 + \frac{\partial x'^1}{\partial x^2} dx^2 + \frac{\partial x'^1}{\partial x^3} dx^3 + \frac{\partial x'^1}{\partial x^4} dx^4$$
$$= \frac{\partial x'^1}{\partial x^J} dx^J.$$

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Generalising this, we get $dx'^i = \dfrac{\partial x'^i}{\partial x^J} dx^J$

This is the transformation law of coordinates.

9. Scalar: A scalar is that quantity which is expressible by one number, eg: Rs 45.25, temperature of a body, mass etc.

10: vector: A vector is that quantity which is expressible by 3 numbers in three dimension. for example the velocity q, is expressible by its 3 components u^1, u^2, u^3.

11. Tensor: A set of quantities $A^{(i)}$ is said to be vector if it satisfies the transformation law

$$A'^{\mu} = A^{\alpha} \frac{\partial x'^{\mu}}{\partial x^{\alpha}} \quad \text{——— (1)}$$

or it satisfies the transformation law

$$A'_{\mu} = A_{\alpha} \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \quad \text{——— (2)}$$

If it satisfies the first one, then it is called Contravariant vector or Contravariant tensor of rank one, and if it satisfies the second one, then it is called covariant vector or covariant tensor of rank one.

The rank of a tensor is defined as a total number of indices or suffixes per component.

The upper position of the suffix is reserved to indicate contravariant character. The lower position of the suffix is reserved to indicate covariant character.

12. Tensor of rank 2: let A_{ij} $(i,j = 1,2,3,\dots,n)$ be n^2 fun of Co-ordinates $x^1, x^2, \dots, x^n$ and if A_{ij} are transformed to A'_{ij} in another coordinate system $x'^1, x'^2, \dots x'^n$ by rule

$A'_{ij} = A_{\alpha\beta} \frac{\partial x^{\alpha}}{\partial x'^{i}} \frac{\partial x^{\beta}}{\partial x'^{j}}$. Then A_{ij} are said to be Covariant tensor of rank two. Similarly if A^{ij} are transformed to A'^{ij}. Then $A'^{ij} = A^{\alpha\beta} \frac{\partial x'^{i}}{\partial x^{\alpha}} \frac{\partial x'^{j}}{\partial x^{\beta}}$ [th]en A^{ij} are called Contravariant tensor of rank 2.