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Contd. of point (12):

If A^i_j ($i, j = 1, 2, 3, \dots, n$) be n^2 functions of coordinates x^1, x^2, \dots, x^n and if A^i_j is transformed to A'^i_j in another coordinate system x'^1, x'^2, \dots, x'^n by rule

$$A'^i_j = A^\alpha_\beta \frac{\partial x'^i}{\partial x^\alpha} \frac{\partial x^\beta}{\partial x'^j} \text{ . Then } A'^i_j \text{ are said to}$$

be components of mixed tensor of rank 2.

13. Tensor of Higher order:

(i) Covariant tensor of rank p .

$$A_{i_1 i_2 \dots i_p} = A_{\alpha_1 \alpha_2 \dots \alpha_p} \frac{\partial x^{\alpha_1}}{\partial x'^{i_1}} \frac{\partial x^{\alpha_2}}{\partial x'^{i_2}} \dots \frac{\partial x^{\alpha_p}}{\partial x'^{i_p}}$$

(ii) Contravariant tensor of rank p .

$$A^{i_1 i_2 \dots i_p} = A^{\alpha_1 \alpha_2 \dots \alpha_p} \frac{\partial x'^{i_1}}{\partial x^{\alpha_1}} \frac{\partial x'^{i_2}}{\partial x^{\alpha_2}} \dots \frac{\partial x'^{i_p}}{\partial x^{\alpha_p}}$$

(iii) Mixed tensor of rank $l+m$

$$A^{i_1 i_2 \dots i_l}_{j_1 j_2 \dots j_m} = A^{\alpha_1 \alpha_2 \dots \alpha_l}_{\beta_1 \beta_2 \dots \beta_m} \frac{\partial x'^{i_1}}{\partial x^{\alpha_1}} \frac{\partial x'^{i_2}}{\partial x^{\alpha_2}} \dots \frac{\partial x'^{i_l}}{\partial x^{\alpha_l}} \frac{\partial x^{\beta_1}}{\partial x'^{j_1}} \frac{\partial x^{\beta_2}}{\partial x'^{j_2}} \dots \frac{\partial x^{\beta_m}}{\partial x'^{j_m}}$$

14. Symmetric tensor: A tensor $A_{\mu\nu}$ is said to be symmetric tensor if $A_{\mu\nu} = A_{\nu\mu}$ — (1)

Property (1) Symmetric property remains unchanged by tensor law of transformation.

Proof: If we show that $A'_{\mu\nu} = A'_{\nu\mu}$.

By (1) $A_{\alpha\beta} = A_{\beta\alpha}$ — (2)

$$A'_{\mu\nu} = A_{\alpha\beta} \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} = A_{\beta\alpha} \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} \text{ by (2)}$$

$$A'_{\mu\nu} = A_{\beta\alpha} \frac{\partial x^\beta}{\partial x'^\mu} \frac{\partial x^\alpha}{\partial x'^\nu} = A'_{\nu\mu} \text{ Proved.}$$

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Property ② A symmetric tensor $A_{\mu\nu}$ has $\frac{4(4+1)}{2}$ independent components.

$A_{\mu\nu}$ has 4^2 independent components in 4 dimensions which are written as

$$\begin{array}{cccc} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{array}$$

No. of component corresponding to a repeated suffix is 4. No. of components corresponding to distinct suffixes is $4^2 - 4$. Due to symmetric property this number is reduced to $\frac{4^2 - 4}{2}$.

\therefore Total number of independent components is $\frac{4^2 - 4}{2} + 4 = \frac{4^2 - 4 + 2 \times 4}{2} = \frac{4^2 + 4}{2} = \frac{4(4+1)}{2}$ Proved.

Note 1: A symmetric tensor $A_{\mu\nu}$ has $\frac{n(n+1)}{2}$ independent components in n dimensions.

Note 2: A tensor $A_{\mu\nu\sigma}$ is said to be symmetric in suffixes μ and ν if $A_{\mu\nu\sigma} = A_{\nu\mu\sigma}$.

This tensor has $\frac{n(n+1)}{2} \cdot n = \frac{n^2}{2}(n+1)$ independent components in n dimensions.

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