$$\therefore L\{\sin\sqrt{t}\} = \frac{\Gamma(3/2)}{p^{3/2}} - \frac{1}{\lfloor 3}\frac{\Gamma(\frac{5}{2})}{p^{5/2}} + \frac{1}{\lfloor 5}\frac{\Gamma(\frac{7}{2})}{p^{7/2}} - \cdots$$

$$= \frac{\frac{1}{2}\Gamma(\frac{1}{2})}{p^{3/2}} - \frac{1}{6}\frac{\frac{3}{2}\frac{1}{2}\Gamma(\frac{1}{2})}{p^{5/2}} + \frac{1}{120}\frac{\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\Gamma(\frac{1}{2})}{p^{7/2}} - \cdots$$

$$= \frac{\frac{1}{2}\sqrt{\pi}}{p^{3/2}} - \frac{1}{8}\frac{\sqrt{\pi}}{p^{5/2}} + \frac{1}{64}\frac{\sqrt{\pi}}{p^{7/2}} - \cdots$$

$$= \frac{\sqrt{\pi}}{2}\frac{1}{p^{3/2}}\left[1 - \frac{1}{4}\frac{1}{p} + \frac{1}{32}\frac{1}{p^2} - \cdots\right]$$

$$= \frac{\sqrt{\pi}}{2}\frac{1}{p^{3/2}}\left[1 - \frac{1}{4p} + \frac{1}{\lfloor 2}\left(\frac{1}{4p}\right)^2 - \cdots\right]$$

$$= \frac{\sqrt{\pi}}{2}\frac{1}{p^{3/2}} e^{-1/4p}$$

—x—

(6) Find the Laplace transform of the function
$F(t) = (\sin t - \cos t)^2$

Ans:- We have,
$F(t) = (\sin t - \cos t)^2$

$$\therefore L\{F(t)\} = L\{\sin t - \cos t\}^2$$

$$= L\{\sin^2 t + \cos^2 t - 2\sin t\cos t\}$$

$$= L\{\sin^2 t\} + L\{\cos^2 t\} - L\{\sin 2t\}$$

$$= L\left\{\frac{1-\cos 2t}{2}\right\} + L\left\{\frac{1+\cos 2t}{2}\right\} - L\{\sin 2t\}$$

$$= \frac{1}{2}\left(\frac{1}{p} + \frac{1}{p}\right) - \frac{1}{2}\left[\frac{p}{p^2+4} - \frac{p}{p^2+4}\right] - \frac{2}{p^2+4} \quad \text{if } p>2$$

$$= \cancel{\frac{1}{p} - \frac{1}{2}\left(\frac{p}{p^2+4}\right) - \frac{2}{p^2+4}}$$

$$= \frac{1}{p} - \frac{2}{p^2+4}$$

$$= \frac{p^2+4-2p}{} \quad \text{if } p>0$$

(7) Find the Laplace transform of the function
$$F(t) = \frac{e^{at}-1}{a}$$

Ans:-
$$L\{F(t)\} = L\left\{\frac{e^{at}-1}{a}\right\}$$
$$= \frac{1}{a}L\{e^{at}\} - \frac{1}{a}L\{1\}$$
$$= \frac{1}{a}\frac{1}{p-a} - \frac{1}{a}\frac{1}{p} = \frac{1}{a}\frac{\cancel{p}-\cancel{p}+a}{p(p-a)} \quad \text{if } p>a$$
$$= \frac{1}{p(p-a)} \quad \text{if } p>a$$

(8) Evaluate $L\{F(t)\}$ if
$$F(t) = \begin{cases} (t-1)^2, & t>1 \\ 0, & 0<t<1 \end{cases}$$

Ans:-
Here $F(t)$ is not defined on $t=0$ and $t=1$
$$\therefore L\{F(t)\} = \int_0^\infty e^{-pt} F(t)\,dt$$
$$= \int_0^1 e^{-pt}\cdot 0\,dt + \int_1^\infty e^{-pt}\cdot(t-1)^2\,dt$$
$$= \int_1^\infty e^{-pt}(t-1)^2\,dt$$
$$= \left[(t-1)^2\frac{e^{-pt}}{-p}\right]_1^\infty - \int_1^\infty 2(t-1)\frac{e^{-pt}}{-p}\,dt$$
$$= -\frac{1}{p}\lim_{t\to\infty}(t-1)^2 e^{-pt} - 0 + \frac{2}{p}\int_1^\infty e^{-pt}(t-1)\,dt$$

$$\therefore L\{F(t)\} = -\frac{1}{p}\lim_{t\to\infty}\frac{(t-1)^2}{e^{pt}} + \frac{2}{p}\left[(t-1)\frac{e^{-pt}}{-p} - \int 1\cdot\frac{e^{-pt}}{-p}dt\right]_1^{\infty}$$

$$= -\frac{1}{p}\lim_{t\to\infty}\frac{2(t-1)}{(pe^{pt})} - \frac{2}{p^2}\left[\frac{(t-1)}{e^{pt}}\right]_1^{\infty} + \frac{2}{p^2}\int_1^{\infty} e^{-pt}dt$$

[By LH Rule]

$$= -\frac{1}{p}\lim_{t\to\infty}\frac{2}{p^2 e^{pt}} - \left[\frac{2}{p^2}\lim_{t\to\infty}\frac{1}{pe^{pt}} - 0\right] + \frac{2}{p^2}\left[\frac{e^{-pt}}{-p}\right]_1^{\infty}$$

[By LH Rule]

$$= 0 - 0 + 0 - \frac{2}{p^3}\left[0 - e^{-p}\right]$$

$$= \frac{2}{p^3}e^{-p} \quad \text{Ans}$$

—x—

(9) Find $L\{F(t)\}$ if $F(t) = \begin{cases} e^t & ; \quad 0<t<5 \\ 3 & ; \quad t>5 \end{cases}$

Ans:- Here $F(t)$ is not defined on $t=0$ and $t=5$

$$\therefore L\{F(t)\} = \int_0^{\infty} e^{-pt}\cdot F(t)dt$$

$$= \int_0^5 e^{-pt}\cdot e^t dt + \int_5^{\infty} e^{-pt}\cdot 3\, dt$$

$$= \int_0^5 e^{-(p-1)t} + 3\int_5^{\infty} e^{-pt}dt$$

$$= \left[\frac{e^{-(p-1)t}}{-(p-1)}\right]_0^5 + 3\left[\frac{e^{-pt}}{-p}\right]_5^{\infty}$$

$$= \frac{e^{-5(p-1)}}{-(p-1)} + \frac{1}{(p-1)} + 3\left[0 + \frac{e^{-5p}}{p}\right]$$

$$= -\frac{e^{-5(p-1)}}{(p-1)} + \frac{1}{p-1} + \frac{3}{p}e^{-5p} \quad \text{Ans}$$

(10) Find $L\{F(t)\}$ · if $F(t) = \begin{cases} t & , 0<t<4 \\ 5 & , t>4 \end{cases}$

Ans:-

Here $F(t)$ is not defined on $t=0$ and $t=4$

$$\therefore L\{F(t)\} = \int_0^{\infty} e^{-pt} . F(t) dt$$

$$= \int_0^4 e^{-pt} . t\, dt + \int_4^{\infty} e^{-pt} . 5\, dt$$

$$= \left[t \frac{e^{-pt}}{-p}\right]_0^4 - \int_0^4 1 . \frac{e^{-pt}}{-p} dt + 5\left[\frac{e^{-pt}}{-p}\right]_4^{\infty}$$

$$= -\frac{1}{p}\left[4 e^{-4p} - 0\right] + \frac{1}{p}\int_0^4 e^{-pt} dt - \frac{5}{p}\left[0 - e^{-4p}\right]$$

$$= \frac{-4e^{-4p}}{p} + \frac{1}{p}\left[\frac{e^{-pt}}{-p}\right]_0^4 + \frac{5}{p} e^{-4p}$$

$$= -\frac{4e^{-4p}}{p} - \frac{e^{-p4}}{p^2} + \frac{5}{p} e^{-4p} + \frac{1}{p^2}$$

$$= \frac{e^{-4p}}{p} - \frac{1}{p^2} e^{-4p} + \frac{1}{p^2}$$

$\rightarrow$ -

(11) $L\{\sin^3 2t\}$

Ans:-

$$\therefore L\{\sin^3 2t\} = L\left\{\frac{3\sin 2t - \sin 6t}{4}\right\}$$

$$= \frac{3}{4} L\{\sin 2t\} - \frac{1}{4} L\{\sin 6t\}$$

$$= \frac{3}{4} \frac{2}{p^2+4} - \frac{1}{4} \frac{6}{p^2+36}$$

$$\therefore L\{\sin^3 2t\} = \frac{6}{4}\left[\frac{1}{p^2+4} - \frac{1}{p^2+36}\right]$$

$$= \frac{6}{4}\left[\frac{\cancel{p^2}+36-\cancel{p^2}-4}{(p^2+4)(p^2+36)}\right]$$

$$= \frac{6}{4}\,\frac{32}{(p^2+4)(p^2+36)}$$

$$= \frac{48}{(p^2+4)(p^2+36)} \quad \text{Ans}$$

—x—

(12) $L\{\cos^3 t\}$

Ans:-

$$\therefore L\{\cos^3 t\} = L\left\{\frac{\cos 3t - 3\cos t}{4}\right\}$$

$$= \frac{1}{4}L\{\cos 3t\} - \frac{3}{4}L\{\cos t\}$$

$$= \frac{1}{4}\frac{p}{p^2+9} - \frac{3}{4}\frac{p}{p^2+1}$$

$$= \frac{p}{4}\left(\frac{p^2+1-3p^2-27}{(p^2+9)(p^2+1)}\right) = \frac{p}{4}\left[\frac{-2p^2-26}{(p^2+9)(p^2+1)}\right]$$

$$= -\frac{1}{2}\,\frac{p^3+13p}{(p^2+9)(p^2+1)} \quad \text{Ans}$$

—x—

*(13) Prove that $L\left\{\frac{1}{\sqrt{\pi t}}\right\} = \frac{1}{\sqrt{p}}$

Ans:- $\because L\{F(t)\} = \int_0^\infty e^{-pt}\cdot F(t)\,dt$

$$\therefore L\left\{\frac{1}{\sqrt{\pi t}}\right\} = \int_0^\infty e^{-pt}\cdot\frac{1}{\sqrt{\pi t}}\,dt$$