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Tensor Calculus (6)

Theorem (4): Prove that tensor (mixed tensor) law of transformation possesses group property.

Proof: Let us consider a transformation of coordinates

$$x^{\mu} \longrightarrow x'^{\mu} \longrightarrow x''^{\mu}$$
$$(i) \longrightarrow (ii) \longrightarrow (iii)$$
$$A^{\mu}_{\nu} \qquad A'^{\mu}_{\nu} \qquad A''^{\mu}_{\nu}$$

In case of transformation (i) $\longrightarrow$ (ii), we have

$$A'^{p}_{q} = A^{\alpha}_{\beta} \frac{\partial x'^{p}}{\partial x^{\alpha}} \frac{\partial x^{\beta}}{\partial x'^{q}} \qquad \text{---} \quad (1)$$

In case of transformation (ii) $\longrightarrow$ (iii), we have

$$A''^{\mu}_{\nu} = A'^{p}_{q} \frac{\partial x''^{\mu}}{\partial x'^{p}} \frac{\partial x'^{q}}{\partial x''^{\nu}} = A^{\alpha}_{\beta} \frac{\partial x'^{p}}{\partial x^{\alpha}} \frac{\partial x^{\beta}}{\partial x'^{q}} \frac{\partial x''^{\mu}}{\partial x'^{p}} \frac{\partial x'^{q}}{\partial x''^{\nu}} \text{ by } (1)$$

$$A''^{\mu}_{\nu} = A^{\alpha}_{\beta} \frac{\partial x''^{\mu}}{\partial x^{\alpha}} \frac{\partial x^{\beta}}{\partial x''^{\nu}}$$

From this it follows that, if we make direct transformation (i) $\longrightarrow$ (iii), then we get the same law of transformation. Hence tensor law possesses group property.

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Definition: Addition of tensors: Two tensors can be added or subtracted provided they are of the same rank and similar character.

Theorem (5): Show that the sum of two tensors is a tensor of the same rank and similar character.

Proof: Let $A^{\sigma}_{\mu\nu}$ and $B^{\sigma}_{\mu\nu}$ are two mixed tensors.

Let their sum is defined as $A^{\sigma}_{\mu\nu} + B^{\sigma}_{\mu\nu} = C^{\sigma}_{\mu\nu}$ —— (1)

If we show that $C^{\sigma}_{\mu\nu}$ is a mixed tensor of rank three, the result will follow.

By (1) $C^{\gamma}_{\alpha\beta} = A^{\gamma}_{\alpha\beta} + B^{\gamma}_{\alpha\beta}$ —— (2)

and
$$C'^{\sigma}_{\mu\nu} = A'^{\sigma}_{\mu\nu} + B'^{\sigma}_{\mu\nu}$$
$$= A^{\gamma}_{\alpha\beta}\frac{\partial x^{\alpha}}{\partial x'^{\mu}}\frac{\partial x^{\beta}}{\partial x'^{\nu}}\frac{\partial x'^{\sigma}}{\partial x^{\gamma}} + B^{\gamma}_{\alpha\beta}\frac{\partial x^{\alpha}}{\partial x'^{\mu}}\frac{\partial x^{\beta}}{\partial x'^{\nu}}\frac{\partial x'^{\sigma}}{\partial x^{\gamma}}$$
$$= \left(A^{\gamma}_{\alpha\beta} + B^{\gamma}_{\alpha\beta}\right)\frac{\partial x^{\alpha}}{\partial x'^{\mu}}\frac{\partial x^{\beta}}{\partial x'^{\nu}}\frac{\partial x'^{\sigma}}{\partial x^{\gamma}}$$
$$\Rightarrow C'^{\sigma}_{\mu\nu} = C^{\gamma}_{\alpha\beta}\frac{\partial x^{\alpha}}{\partial x'^{\mu}}\frac{\partial x^{\beta}}{\partial x'^{\nu}}\frac{\partial x'^{\sigma}}{\partial x^{\gamma}} \quad \text{by (2)}$$

This proves the required result.

—x—

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