

## Change of scale property

Theorem: If  $L\{F(t)\} = f(p)$  then  $L\{F(at)\} = \frac{1}{a} f(p/a)$

Proof: By definition we have

$$L\{F(at)\} = \int_0^{\infty} e^{-pt} F(at) dt$$

Putting  $at = x$

$$a dt = dx$$

limit does not change

$$\therefore L\{F(at)\} = \frac{1}{a} \int_0^{\infty} e^{-\frac{p}{a}x} F(x) dx$$

$$= \frac{1}{a} \int_0^{\infty} e^{-\frac{p}{a}x} F(x) dx$$

$$= \frac{1}{a} \int_0^{\infty} e^{-\frac{p}{a}t} F(t) dt \quad \left\{ \because \int_a^b f(x) dx = \int_a^b f(t) dt \right\}$$

$$= \frac{1}{a} f\left(\frac{p}{a}\right) \quad \left\{ \because f(p) = \int_0^{\infty} e^{-pt} F(t) dt \right\}$$

— x —

## Problems

- (1) Applying change of scale property, obtain the Laplace transform of  $\sinh 3t$ .

Ans:

$$L\{\sinh t\} = \frac{1}{p^2 - 1} = f(p) \quad (\text{say})$$

$$\therefore L\{\sinh 3t\} = \frac{1}{3} f\left(\frac{p}{3}\right)$$

$$= \frac{1}{3} \frac{1}{\frac{p^2}{9} - 1} = \frac{1}{3} \frac{9}{p^2 - 9}$$

$$= \frac{3}{p^2 - 9} \quad \text{Ans.}$$

② Find  $L\{\cos 5t\}$  by change and scale property.

Ans:-

$$L\{\cos t\} = \frac{p}{p^2+1} = f(p) \text{ (say)}$$

$$\therefore L\{\cos 5t\} = \frac{1}{5} f(p/5)$$

$$= \frac{1}{25} \frac{p/5}{(p/5)^2+1}$$

$$= \frac{1 \cdot p}{25 \frac{p^2+25}{25}} = \frac{25p}{p^2+25} \times \frac{1}{25}$$

$$= \frac{p}{p^2+25}$$

—x—

Given  $L\{F(t)\} = \frac{p^2-p+1}{(2p+1)^2(p-1)}$ , applying the

change of scale property show that

$$L\{F(2t)\} = \frac{p^2-2p+4}{4(p+1)^2(p-2)}$$

We have,

$$L\{F(t)\} = \frac{p^2-p+1}{(2p+1)^2(p-1)} = f(p) \text{ (say)}$$

$$\therefore L\{F(2t)\} = \frac{1}{2} f(p/2)$$

$$= \frac{1}{2} \frac{\left(\frac{p}{2}\right)^2 - \frac{p}{2} + 1}{\left(2\frac{p}{2}+1\right)^2\left(\frac{p}{2}-1\right)}$$

$$= \frac{1}{2} \frac{\frac{p^2}{4} - \frac{p}{2} + 1}{(p+1)^2(p-2)}$$

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$$\begin{aligned}
 \therefore L\{F(2t)\} &= \frac{1}{2} \frac{p^2 - 2p + 4}{(p+1)^2(p-2)} \\
 &= \frac{1}{2} \frac{p^2 - 2p + 4}{(p+1)^2(p-2)} \\
 &= \frac{1}{4} \frac{p^2 - 2p + 4}{(p+1)^2(p-2)} \quad \text{proved.}
 \end{aligned}$$

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④

If  $L\{F(t)\} = \frac{1}{p} e^{-1/p}$ , prove that

$$L\{e^{-t} F(3t)\} = \frac{e^{-3/(p+1)}}{(p+1)}$$

$$\therefore L\{F(3t)\} = \frac{1}{3} f(p/3)$$

[By change and scale prop]

$$= \frac{1}{3} \frac{1}{p/3} \cdot e^{-1/(p/3)}$$

$$= \frac{1}{3} \times \frac{3}{p} e^{-3/p}$$

$$= \frac{1}{p} e^{-3/p} = f(p) \quad (\text{say})$$

$\therefore$  By 1st shifting theorem we have,

$$L\{e^{-t} F(3t)\} = f(p+1)$$

$$= \frac{1}{(p+1)} e^{-3/(p+1)}$$

$$= \frac{e^{-3/(p+1)}}{(p+1)}$$

$$(p+1)$$

Ans.