Complement of DN form

Complement of a Boolean expression in DN form is the Boolean expression which is formed by sum of exactly those terms of complete D.N form which are missing in given D.N form.

For example complement of $xy + x'y$ is $xy' + x'y'$

Conjunctive Normal Form (CNF)

A Boolean expression is said to be in CNF in n variables $x_1, x_2, \dots x_n$ $(n>1)$ if the expression is of the form of a product of factors in which each factor is of the type $f_1(x_1) + f_2(x_2) + \dots + f_n(x_n)$ where

$$f_i(x_i) = x_i \text{ or } x_i' \quad \text{for } \forall \; i = 1, 2, 3 \dots n.$$

and no two factors are same

Example (i) $(x+y)\cdot(x+y')\cdot(x'+y)$ is a CNF in two variables

(ii) $(x+y+z)\cdot(x+y'+z)\cdot(x'+y'+z)$ is a CNF in three variables

EXAMPLES

(1) Express the Boolean expression $(x+y+z)(xy+x'z)'$ in CNF.

Ans

$$(x+y+z)\cdot(xy+x'z)'$$

$$=(x+y+z)\cdot\{(xy)'\cdot(x'z)'\}$$

$$=(x+y+z)\cdot(x'+y')\cdot(x+z')$$

$$=(x+y+z)\cdot(x'+y'+(z\cdot z'))\cdot(x+z'+(yy'))$$

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$= (x+y+z)(x'+y'+z)\cdot(x'+y'+z')(x+z'+y)(x+z+y')$

$= (x+y+z)(x'+y'+z)(x'+y'+z')(x+y+z')(x+y'+z)$
(which is CNF)

Complete Conjuctive Normal form.

A C.N.F. to n variables which contains 2^n factors is called complete CNF.

Thm A complete CNF is identically

EQUIVALENT:

Defn: Two Boolean expressions are said to be equivalent if and only if their respective CNFs contain same factors.

Complement of a Boolean Expression in CNF :-

The complement of a Boolean expression in CNF is the Boolean expression formed by product of exactly those factors of complete CNF missing in given CNF.

Transformation from one normal form to other

We use the fact that complement of complement is original expression. ie if f is Boolean expression then $(f')' = f$.

ie We take complement of one normal form and the find its complement.

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Example

(1) Transform DNF $x'y + xy' + x'y'$ into an expression in CNF.

Solⁿ Let $f = x'y + xy' + x'y'$

It's complement is

$f' = xy$ [$\because$ Complete DNF is $xy + x'y + xy' + x'y'$]

Taking its complement

$(f')' = (xy)'$

i.e. $f = x' + y'$. (By De-Morgan's law).

Theorem: (Boole's Expansion theorem)

Any function $f(a, b, c, \ldots n)$ can be expressed as

$$f(a,b,c,\ldots n) = f(1,1,1,\ldots 1)abc\ldots n + f(0,1,1,\ldots 1)a'bc\ldots n$$
$$+ f(1,0,1,\ldots 1)ab'c\ldots n + f(1,1,0,\ldots 1)abc'\ldots n$$
$$\ldots + f(0,0,0\ldots 0)a'b'c'\ldots n'$$

Proof: We know that every function can be expressed in additive normal form so let us suppose that

$$f(a,b,c,\ldots n) = \alpha_1 abc\ldots n + \alpha_2 a'bc\ldots n + \alpha_3 ab'c\ldots n$$
$$+ \alpha_4 abc'\ldots n + \ldots + \alpha_n a'b'c'\ldots n'$$

where α_i takes values either 0 or 1 $\forall i$ — (1)

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We also know that only one term of the function in additive normal form will assume value 1 for any arbitrary allotment of the values 0 & 1 to the variables. Let

Let a particular allotments of 0 & 1 to the variables in LHS of eqn(1) has value 1. and RHS of eqn(1) becomes $\alpha(1)$.

By equality of ~~of~~ two functions

$$1 = \alpha \cdot 1$$

$$\alpha = 1$$

With similar argument on arbitrary assignment of 0 or 1 to these variable we will be able to evaluate all α_i.

Putting these values of α_i in RHS of eqn(1) we get the Boolean expansion of the function.

Problems for Practice

(1) Write in D.N.F. & CNF

(a) $F(x,y,z) = [(x'+y')+(y+z')']' + yz$

(b) $F(x,y,z) = [x+(x'+y')'][x+(y'z')']$

(c) $F(x,y,z) = (x+y)(y+z)+(y'+z)$

(d) $F(x,y) = (xy'+xy)' + x'$

(e) $F(x,y,z) = (x+y+z)(xy+x'z)'$