

Brief introduction of Thermal radiation

Part III

Stefan's Law:- It states that the total amount of energy radiated per second per unit area of a perfect black body is directly proportional to the fourth-power of the absolute temperature of the surface of the body. i.e.

$$E \propto T^4$$

$$E = \sigma T^4$$

where σ is called the Stefan's Constant. Its value is $5.67 \times 10^{-8} \text{ J s}^{-1} \text{ m}^{-2} \text{ K}^{-4}$.

If a perfect black at temperature T is placed in an enclosure at temperature T_0 , then heat-energy radiated per second per unit area is given by

$$E = \sigma (T^4 - T_0^4)$$

If the body is not a perfect black body, the amount of heat-energy radiated per second per unit area will be given by

$$E = e\sigma (T^4 - T_0^4)$$

where 'e' is called emissivity of the surface. Its value depends upon nature of the surface.

This law is also known as Stefan-Boltzmann Law.

Newton's Law of Cooling: This Law states that the rate of cooling of a body is directly proportional to the temperature difference between the body and the surroundings, provided the temperature difference is small.

Deduction of Newton's Law of Cooling from Stefan-Boltzmann Law.

Let T_0 be the temperature of surroundings and ΔT be the ^{small} temperature difference between the surrounding and a body, then the temperature of the body is given by

$$(T = T_0 + \Delta T)$$

By using Stefan's - Boltzmann Law, we may write

$$E = A\sigma e[(T_0 + \Delta T)^4 - T_0^4]$$

$$E = A\sigma T_0^4 \left[\left(1 + \frac{\Delta T}{T_0}\right)^4 - 1 \right]$$

By using binomial theorem

$$E = A\sigma T_0^4 \left[1 + 4 \frac{\Delta T}{T_0} + \text{term containing higher powers of } \frac{\Delta T}{T_0} - 1 \right]$$

$$E = A\sigma T_0^4 \left[\left(1 + \frac{4\Delta T}{T_0}\right) - 1 \right] \quad \text{Neglect higher power terms}$$

$$E = 4A\epsilon\sigma T_0^3 \Delta T$$

Since $4A\epsilon\sigma T_0^3$ is constant - (K) Let-

$$E = K \Delta T$$

$$E = K (T - T_0)$$

$$\therefore T = T_0 + \Delta T$$

$$\Rightarrow E \propto (T - T_0)$$

This is Newton's Law of Cooling