MODAL QUESTIONS
of

U. G. Sem – VI , Paper – CC 613

PREPARED
BY

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For, KOLHAN UNIVERSITY, CHAIBASA

Time : 3 Hours
FULL MARKS : 70

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

PART A

Q. No. 1. Answer all the questions $2 \times 10 = 20$

(a) If a suffix occurs twice in a term, once in upper position and once in lower position then that suffix is called

(i) real suffix (ii) dummy suffix
(iii) imaginary suffix (iv) None of these.

(b) Kronecker delta, it is denoted by δ_{ij} is defined as

(i) $\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$ (ii) $\delta_{ij} = \begin{cases} -1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$

(iii) $\delta_{ij} = \begin{cases} \infty & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$ (iv) $\delta_{ij} = \begin{cases} 0 & \text{if } i = j \\ 1 & \text{if } i \neq j \end{cases}$

(c) A skew symmetric tensor $A^{\lambda\mu}$ in n-dimensions has

(i) $\frac{n-1}{2}$ independent components

(ii) $\frac{n+1}{2}$ independent components

(iii) $\frac{n(n-1)}{2}$ independent components

(iv) $\frac{n(n+1)}{2}$ independent components

(d) Kronecker delta is a mixed tensor of

(i) rank 1 (ii) rank 0 (iii) rank ∞ (iv) rank two

(e) If ϕ is an invariant then $\frac{\partial \phi}{\partial x^i}$ is a

(i) mixed vector (ii) Covariant vector

(iii) Contravariant vector (iv) Null vectors.

(f) The infinite Fourier transform of $F(x)$ is denoted by $F\{F(x)\}$ and is defined as

(i) $F\{F(x)\} = \int_{-\infty}^{\infty} F(x)\, e^{-isx}\, dx, \quad -\infty < x < \infty$

(ii) $F\{F(x)\} = \int_{-\infty}^{\infty} F(x)\, e^{-sx}\, dx, \quad -\infty < x < \infty$

(iii) $F\{F(x)\} = \int_{-\infty}^{\infty} F(x)\, e^{isx}\, dx, \quad -\infty < x < \infty$

(iv) $F\{F(x)\} = \int_{-\infty}^{\infty} F(x)\, e^{-sx}\, dx, \quad -\infty < x < \infty$

(g) The fourier sine tranform of e^{-x} is

(i) $\frac{1}{1+p^2}$ (ii) $-\frac{1}{1+p^2}$ (iii) $\frac{2}{1+p^2}$ (iv) $\frac{p}{1+p^2}$

(h) $F(x) = F^{-1}\{f(s)\}$ is equal to

(i) $\frac{1}{2\pi}\int_{-\infty}^{\infty} f(s)\, e^{isx}\, ds$ (ii) $\frac{1}{2\pi}\int_{-\infty}^{\infty} f(s)\, e^{-isx}\, ds$

(iii) $\frac{1}{2\pi}\int_{-\infty}^{\infty} f(s)\, e^{-sx}\, ds$ (iv) $\frac{1}{2\pi}\int_{-\infty}^{\infty} f(s)\, e^{-sx}\, ds$

(i) The finite tranform of $f(x)=1$, where $\alpha x < \pi$ is

(i) $\frac{\pi\,(-1)^{p+1}}{p}$ (ii) $\frac{(-1)^{p-1}}{p^2}$

(iii) $1 + \frac{(-1)^{p+1}}{p}$ (iv) $\frac{(-1)^{p+1}}{p^2}$

(j) If $F\{F(x)\} = f(s)$, then $F\left\{\frac{d^n F}{dx^n}\right\}$ is equal to

(i) $(s)^n f(s)$ (ii) $s^n f(s)$ (iii) $s^n f^n(s)$ (iv) None of these.

PART - B

Answer any four questions of the following

$5 \times 4 = 20$

Q.No.2. Prove that the outer product of two tensors is a tensor.

Q.No.3. Show that $\sqrt{g}\, dx^1 dx^2 \ldots dx^n$ is an invariant.

Q. No. (4) Define Covariant derivative of a contravariant vector and show that it is a tensor

Q. No. 5. If $A_{iJ} = A_{i,J} - A_{J,i}$. Prove that
$$A_{iJ,K} + A_{JK,i} + A_{Ki,J} = 0$$

Q. No. 6. Show That $F\left\{\frac{d^nF}{dx^n}\right\} = (is)^n f(s)$,
where $F\{F(x)\} = f(s)$.

Q. No. 7. Find the fourier transform of
$$f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$$

Q. No. 8. Using Fourier Sine integral formula, Prove that
$$\int_0^{\infty} \left\{\frac{1-\cos(\pi\lambda)}{\lambda}\right\} \sin(x\lambda)\, d\lambda = \begin{cases} \frac{\pi}{2}, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$$

Q. No. 9. Find fourier transform of $f(x)$, defined by
$$f(x) = \begin{cases} 1, & |x| \leq a \\ 0, & |x| > a \end{cases}$$
and hence prove that $\int_0^{\infty} \frac{\sin^2 ax}{x^2}\, dx = \frac{\pi a}{2}$.

PART – C

Answer any two of the following:

$15 \times 2 = 30$

Q. No. 10. State and Prove Quotient law of tensors.

Q. No. 11. Prove that

(i) $\Gamma_{iJ,K} + \Gamma_{JK,i} = \dfrac{\partial g_{iK}}{\partial x^J}$

(ii) $\Gamma^{i}_{iJ} = \dfrac{\partial}{\partial x^J} \log \sqrt{(-g)}$

(iii) $\Gamma^{i}_{iJ} = \dfrac{\partial}{\partial x^J} \log \sqrt{g}$

(iv) $\Gamma \dfrac{\partial g^{iJ}}{\partial x^K} = -g^{il}\,\Gamma^{J}_{lK} - g^{lJ}\,\Gamma^{i}_{lK}$

Q. No. 12. $5 \times 3 = 15$

(a) If c_1, c_2 are arbitrary constants, then Prove that $F\{c_1 F(x) + c_2 G(x)\} = c_1 F\{F(x)\} + c_2 F\{G(x)\}$

(b) If $f(s)$ is the Fourier transform of $F(x)$. Then prove that $\frac{1}{a} f\left(\frac{s}{a}\right)$ is the Fourier transform of $F(ax)$.

(c) If $f(s)$ is the Fourier transform of $F(x)$, then prove that $e^{-ias} f(s)$ is the Fourier transform of $F(x-a)$.

Q. No. 13. State and Prove Fourier Integral Theorem.

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Answers of Objective Q. No. 1

a — (ii)
b — (i)
c — (iii)
d — (iv)
e — (ii)
f — (i)
g — (iv)
h — (i)
i — (iii)
J — (i)

→ x —

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K.C.C.