Model Question Paper

UG, Sem-V, Mathematics Hons.

Paper → DSEMATH-501 A (Hons.)

Kolhan University, Chaibasa

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Model Question Paper of UG, Sem-V, 2020,

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Answer from all the Parts as directed.

The figures in the right-hand margin indicate marks.

Full Marks : 70

Time : 3 hours

Part-A
(Compulsory)

1. Choose the correct answer of the following : $2\times10=20$

(a) A basis of a vector space cannot contain

(i) the zero vector (ii) a non-zero vector

(iii) a negative vector (iv) a positive vector

(b) If V is a finite dimensional and W is a subspace of V, then $\dim\left(\frac{V}{W}\right)$ is

(i) $\dim V - \dim W$ (ii) $\dim V + \dim W$ (iii) $\dim V / \dim W$

(c) Two subspaces W_1 and W_2 of a vector space $V(F)$ are said to be disjoint if

(i) $W_1 \cap W_2 = \varphi$ (ii) $W_1 \cap W_2 = \{0\}$ (iii) $W_1 \cup W_2 = V$

(d) If $f : V_1 \to V_2$, then $\operatorname{Ker} f =$

(i) $\{1\}$ (ii) $\{0\}$ (iii) $\{0,1\}$ (iv) none of these

(e) If V^* is dual space of a finite dimensional vector space V and $x, y \in V$, $x \neq y$, then there is a $f \in V^*$ such that

(i) $f(x) = f(y)$ (ii) $f(x) \neq f(y)$ (iii) $f(x) \geq 0$ (iv) None of these

(f) The general solution of the difference equation $y_{x+2} - 4y_{x+1} + 4y_x = 0$ is

(i) $y_x = (c_1 + c_2 x) 2^x$ (ii) $y_x = (c_1 + c_2 x) 3^x$

(iii) $y_x = c_1 2^x + c_2 3^x$ (iv) $y_x = (c_1 + c_2 x) 5^x$

(g) The order and degree of the equation

$$y_{k+1}^2 \, y_{k+2}^3 - y_{k+1} \, y_k - y_k^2 = k \text{ are}$$

(i) 2 and 3 (ii) 3 and 2 (iii) 3 and 4 (iv) none of these

(h) The 2nd order homogeneous difference equation is

(i) $y_{x+2} + k_1 y_{x+1} + k_2 y_x = x^2$

(ii) $y_{x+2} + k_1 y_{x+1} + k_2 y_x = 0$

(iii) $y_{x+1} + a y_x = 0$ (iv) None of these.

(i) The solution of the diff. equation $y_{x+1} - y_x = x$ is

(i) $y_x = \frac{(x+1)(x+2)}{2}$ (ii) $y_x = \frac{x(x-1)}{2}$ (iii) $y_x = \frac{x(x+1)}{2}$

(j) The degree and order of the difference equation are free from Δ. Is it true or false.

Part-B

Answer any four questions : $5 \times 4 = 20$

2. Let $V(F)$ be a vector space. If $v_1, v_2, \ldots, v_n$ are non-zero vectors, then prove that they are linearly dependent iff some v_k, $2 \le k \le n$ is a linear combination of the preceding ones.

3. Show that the set $A = \{(1,0,-1), (1,1,1), (1,0,0)\}$ is a basis of R^3 where R is the field of real numbers. Hence find the co-ordinates of the vector (a,b,c) relative to above basis.

4. Let V_1 be a subspace of finite dimensional vector space V. Then prove that

$$\dim V = \dim V_1 \iff V = V_1$$

5. Let $T: R^3 \to R^3$ be a transformation defined by

$$T(x,y,z) = (x+2y-z, \; y+z, \; x+y-2z).$$

Verify that T is a linear transformation.

6. Solve $y_{k+1} + 5y_k = 2k$.

7. Show that $y_x = c_1 + c_2 2^x - x$ is a solution of the difference equation $y_{x+2} - 3y_{x+1} + 2y_x = 1$

8. Solve the equation $y_{k+1} + y_k = 3$, $y_0 = 6$ over the set $k = 0, 1, 2, \ldots$

9. Solve $y_{x+2} - 4y_{x+1} + 4y_x = 3^x + 2^x + 4$.

P.T.O.

Part - C

Answer any two questions: $2 \times 15 = 30$

10. (a) Derive the necessary and sufficient condition for a non-empty subset W of a vector space V(F) to be a subspace of V.

(b) Prove that the intersection of arbitrary subspaces of a vector space is a subspace. Is it true for union?

11. (a) State and prove existence theorem on basis of finite dimensional vector space.

(b) Show that the mapping $T: V_2(R) \to V_3(R)$ defined by $T(a,b) = (a+b, a-b, b)$ is a linear transformation from V_2 into V_3. Find the range, rank and nullity of T.

12. (a) Solve $y_{x+2} - 7y_{x+1} + 12y_x = 0$.

(b) Derive the general solution of $y_{x+1} = Ay_x + B$.

13. (a) Solve the diff. equation $y_{K+2} - 3y_{K+1} + 2y_K = a^K$ where a is some constant.

(b) Solve $y_{x+3} - 5y_{x+2} + 8y_{x+1} - 4y_x = x2^x$.

—End—

Remark: Ans. of Q. 1

(a) (i)	(c) (ii)	(e) (ii)	(g) (i)	(i) (ii)
(b) (i)	(d) (ii)	(f) (i)	(h) (ii)	(j) True