MODAL QUESTIONS

of

U. G. sem – V, PAPER – CC 5 11

PREPARED
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For, KOLHAN UNIVERSITY, CHAIBASA

Time : 3 Hours

Full Marks : 70

Candidates are required to give their answers in their own words as far as possible.

The figures in the margin indicate full marks.

PART – A

Q. NO. 1. Answer all questions $2 \times 10 = 20$

(a) If $\vec{T}$ be tension of a string of length L, then work done by the tension $\vec{T}$ is

(i) $T\,dl$ (ii) $-T\,dl$ (iii) $l\,dT$ (iv) $-l\,dT$.

(b) If T is the tension at any point P of a catenary and T_0, that at lowest point C, then the weight W of the arc CP of the catenary is

(i) $2TT_0$ (ii) $\sqrt{T^2-T_0^2}$ (iii) $\sqrt{T^2+T_0^2}$ (iv) None of these

(c) The pitch of the wrench "P" is denoted by the ratio

(i) $\frac{K}{R}$ (ii) $-\frac{K}{R}$ (iii) $\frac{R}{K}$ (iv) $-\frac{R}{K}$

(d) When the upper body of radius r near the point of contact be a plane, then $r \to \infty$. Then the equilibrium is stable if

(i) $h \leq R$ (ii) $h \geq R$ (iii) $h < R$ (iv) $h > R$

(e) The intrinsic equation of the catenary is

(i) $s = c\tan\psi$ (ii) $s = c\sin\psi$ (iii) $s = c\cos\psi$ (iv) $s = c\sec\psi$.

(f) Rate of description of the sectorial area is

(i) $\frac{1}{2} r \frac{d\theta}{dt}$ (ii) $\frac{1}{2} \frac{d\theta}{dt}$ (iii) $\frac{1}{2} r^2 \frac{d\theta}{dt}$ (iv) $\frac{1}{2} r^2 \frac{d^2\theta}{dt^2}$

(g) At an apse, $\frac{dr}{d\theta}$ is equal to

(i) 1 (ii) -1 (iii) ∞ (iv) 0

(h) In elliptic path, if the particle is projected with the V from the centre of force, then

(i) $V^2 = \frac{2\mu}{r}$ (ii) $V^2 < \frac{2\mu}{r}$ (iii) $V^2 > \frac{2\mu}{r}$ (iv) None of these

(1) The Periodic time of a closed central orbit under inverse square law is in the relation

(i) $T^2 > \frac{4\pi^2 a^3}{\mu}$ (ii) $T^2 < \frac{4\pi^2 a^3}{\mu}$ (iii) $T^2 = \frac{4\pi^2 a^3}{\mu}$ (iv) $T^2 < \frac{4\pi a^3}{\mu}$

(J) Latus rectum of the trajectory is equal to

(i) $\frac{2}{g}$ (horizontal velocity) (ii) $\frac{2}{g^2}$ (horizontal velocity)

(iii) $\frac{2}{g}$ (horizontal velocity)^2 (iv) $\frac{2}{\sqrt{g}}$ (horizontal velocity)^2

PART – B

Answer any four questions of the following $5 \times 4 = 20$

Q.No.2. Find the virtual work done by the tension of a string, when the displacement will make the string to vary in length.

Q.No.3. A uniform chain of length l is to be suspended from two points A and B in the same horizontal line, so that either terminal tension is n times that at the lowest point. Show that $\frac{l}{\sqrt{n^2-1}} \log\left(n + \sqrt{n^2-1}\right)$

Q.No.4. The middle points of the opposite sides of a jointed quadrilateral are connected by light rods of length l and l'. If T and T' be the tensions in these rods. Prove that

$$\frac{T}{l} + \frac{T'}{l'} = 0$$

Q.No.5. Find the necessary and sufficient conditions for equilibrium of a rigid body.

Q.No.6. A particle describes the curve $r^n = a^n \cos n\theta$ under a force to the pole. Find the law of force.

Q.No.7. If V_1 and V_2 are the linear velocities of a planet, when it is respectively nearest and farthest from the sun. Prove that

$$(1-e)V_1 = (1+e)V_2.$$

Q.No.8. A particle is projected from O at an elevation α and after t seconds it appears to have an elevation β as seen from the point of projection. Prove that the initial velocity was $\frac{gt\cos\beta}{2\sin(\alpha-\beta)}$.

Q.No.9. Prove that the reversed effective forces acting on each particle of the body and the impressed (external) forces on the system are in equilibrium.

PART – C

Answer any two questions of the following

$15 \times 2 = 30$

Q.No. 10 (a) State and prove that, the Converse of the principle of virtual work.

(b) A body rests in equilibrium upon another fixed body, the portions of the two bodies in contact being spheres of radii R and r respectively and the straight line joining the centres of the spheres being vertical, if the first body be slightly displaced, to find whether the equilibrium is stable or unstable. The bodies being rough enough to prevent sliding.

Q.No. 11. A heavy uniform inextensible string hangs freely under gravity. Find the cartesian equation of catenary and also find the relation between tension T' and ordinate 'y' of any point (x, y) of the common catenary.

Q. No. 12. A particle subject to a force producing an acceleration $\mu\left(\frac{r+2a}{r^5}\right)$ towards the origin is projected from the point (a, 0) with a velocity equal to the velocity from the infinity at an angle $\cot^{-1}2$, with the initial line, Show that the equation of the path is

$$r = a(1+2\sin\theta).$$

Q. No. 13. A particle of mass m is projected in a vertical plane through the point of projection with velocity u in a direction making an angle α with the horizontal, show that the path of the projectile in vaccum is a parabola. Also find the latus rectum, vertex, focus and directrix of the trajectory.

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Answers of objective questions, Q. No. 1

a —— (ii)
b —— (ii)
c —— (i)
d —— (iii)
e —— (i)
f —— (iii)
g —— (iv)
h —— (ii)
i —— (iii)
J —— (iii)

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K. C. C.