Model Question Paper

UG, Sem-V, Mathematics Hons.

Paper: DSEMATH-501B (Hons.)

Kolhan University, Chaibasa

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Model Question Paper of UG, Sem-V, 2020

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Answer from all the Parts as directed.
The figures in the right-hand margin indicate Marks.

Full Marks: 70
Time: 3 hours

Part-A
(Compulsory)

1. Choose the correct answer of the following: $2\times10=20$

(a) The value of i^i is equal to

(i) $e^{-(4n+1)\frac{\pi}{2}}$ (ii) $e^{-(2n+1)\pi}$ (iii) 0 (iv) $e^{2n\pi}$

(b) The Sum of the Series

$\cos\frac{\pi}{11}+\cos\frac{3\pi}{11}+\cos\frac{5\pi}{11}+\cos\frac{7\pi}{11}+\cos\frac{9\pi}{11}$ is

(i) $\frac{1}{2}$ (ii) $\frac{\pi}{2}$ (iii) $\frac{3}{2}$ (iv) 2

(c) If $K_n=\cos\frac{\pi}{2^n}+i\sin\frac{\pi}{2^n}$, then the product $K_1\cdot K_2\cdot K_3\cdot$ ---- ad inf. is equal to

(i) 0 (ii) 1 (iii) -1 (iv) ∞

(d) The Sum of $\theta-\frac{\theta^3}{3}+\frac{\theta^5}{5}-$ ---- to ∞ ; $|\theta|<1$, is

(i) $\sin\theta$ (ii) $\cos\theta$ (iii) $\tan^{-1}\theta$ (iv) $\sin^{-1}\theta$

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(e) The principal value of $\sinh^{-1} w$ is

(i) $\log(w+\sqrt{w^2+1})$ (ii) $\log(w-\sqrt{w^2+1})$ (iii) $\log(1+w)$ (iv) 0

(f) The scalar triple product of $\vec{b}-\vec{c}$, $\vec{c}-\vec{a}$, $\vec{a}-\vec{b}$ is

(i) $2[\vec{a}\ \vec{b}\ \vec{c}]$ (ii) $[\vec{a}\ \vec{b}\ \vec{c}]^2$ (iii) 0 (iv) None of these

(g) $(\vec{a}\cdot\vec{b})\vec{b} + \vec{b}\times(\vec{a}\times\vec{b}) = b^2\vec{a}$ is

(i) True (ii) False

(h) If $\vec{v}$ is a vector function and $\vec{v}\times\frac{d\vec{v}}{dt}=0$, then $\vec{v}$ has

(i) Constant magnitude (ii) Constant direction

(iii) no constant magnitude as well as direction.

(i) If $\vec{V} = x\vec{i}+y\vec{j}+z\vec{k}$, then $\operatorname{div}(\vec{V})$ is equal to

(i) 1 (ii) 3 (iii) 0 (iv) $2\vec{V}$

(j) The normal vector to the surface $x^2+y^2-z=0$ at $(-1,-2,5)$ is

(i) $2\vec{i}+4\vec{j}+\vec{k}$ (ii) $\vec{i}+\vec{j}+\vec{k}$ (iii) $2\vec{i}-\vec{j}$ (iv) $2\vec{k}$

Part-B

Answer any four questions: $4\times5=20$

2. Apply De Moivre's theorem to solve the equation

$$x^7+x^4+x^3+1=0$$

3. If $i^{(\alpha+i\beta)} = \alpha+i\beta$, prove that

$$\tan\frac{(4n+1)\pi\alpha}{2} = \frac{\beta}{\alpha} \quad \text{and} \quad \alpha^2+\beta^2 = e^{-(4n+1)\pi\beta}$$

4. If $\sin(\theta+i\varphi) = u+iv$, prove that $\sin^2\theta$ and $\cosh^2\varphi$ are the roots of the equation $t^2 - t(1+u^2+v^2) + u^2 = 0$.

5. Evaluate $\sum_{r=1}^{n} \tan^{-1}\frac{1}{2r^2}$

6. Prove that $[\vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\ \ \vec{a}\times\vec{b}] = [\vec{a}\ \vec{b}\ \vec{c}]^2$

7. If $\vec{n}$ is an unit vector, prove that

$$\left|\vec{n}\times\frac{d\vec{n}}{dt}\right| = \left|\frac{d\vec{n}}{dt}\right|$$

8. Prove that $\text{curl}(\varphi\vec{a}) = \varphi\,\text{curl}\,\vec{a} + (\text{grad}\,\varphi)\times\vec{a}$.

9. Show that $r^n\vec{r}$ is an irrotational vector for any value of n but it is Solenoidal only if $n=-3$, where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$.

Part-C

Answer any two questions : $2\times 15 = 30$

10. (a) Separate into real and imaginary parts of the expression $(x+iy)^{\alpha+i\beta}$.

(b) If α and β are the roots of $x^2 - 2x + 4 = 0$, prove that $\alpha^n + \beta^n = 2^{n+1}\cos\frac{n\pi}{3}$

11. (a) If $y = \log\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$, prove that

$$x = -i\log\tan\left(\frac{\pi}{4} + i\frac{y}{2}\right).$$

(b) Sum the Series

$$\sin\theta - \frac{\sin 2\theta}{\lfloor 2} + \frac{\sin 3\theta}{\lfloor 3} - \text{------ ad inf.}$$

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12. (a) Show that the necessary and sufficient condition for the vector function $\vec{V}$ of the scalar variable t to have constant magnitude is

$$\vec{V} \cdot \frac{d\vec{V}}{dt} = 0$$

(b) Prove that

$$(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d}) + (\vec{b}\times\vec{c})\cdot(\vec{a}\times\vec{d}) + (\vec{c}\times\vec{a})\cdot(\vec{b}\times\vec{d}) = 0$$

and deduce that

$$\sin(\alpha+\beta)\sin(\alpha-\beta) = \sin^2\alpha - \sin^2\beta.$$

13. (a) If $\vec{R} = \vec{a}\cos\omega t + \vec{b}\sin\omega t$, show that-

(i) $\vec{R}\times\frac{d\vec{R}}{dt} = \omega\,\vec{a}\times\vec{b}$

(ii) $\frac{d^2\vec{R}}{dt^2} = -\omega^2\vec{R}$.

(b) Prove that

$$\text{Div}(\vec{a}\times\vec{b}) = \vec{b}\cdot(\text{curl}\,\vec{a}) - \vec{a}\cdot(\text{curl}\,\vec{b}).$$

— End —

Remark: Answer of Q. 1

(a) (i)
(b) (i)
(c) (iii)
(d) (iii)
(e) (i)
(f) (iii)
(g) (i)
(h) (ii)
(i) (ii)
(j) (i)