# The Inverse Laplace Transform



Inverse Laplace Transform (Defn): If $f(p)$ is the Laplace Transform of a function $F(t)$

i.e; $L\{F(t)\} = f(p)$

then $F(t)$ is called the inverse Laplace transform of the function $f(p)$ and is written as

$$F(t) = L^{-1} f(p)$$

$L^{-1}$ is called the inverse Laplace transformation operator.

## Problems (Formula)

(1) Find $L^{-1}\left(\frac{1}{p}\right)$

Ans:

$\because L\{1\} = \frac{1}{p}$

$\therefore L^{-1}\left\{\frac{1}{p}\right\} = 1$

(2) Find $L^{-1}\left\{\frac{1}{p^{n+1}}\right\}$, $n$ is a +ve integer

Ans:-

$\because L\{t^n\} = \frac{\lfloor n}{p^{n+1}}$

$\Rightarrow L^{-1}\left\{\frac{1}{p^{n+1}}\right\} = \frac{t^n}{\lfloor n}$

(3) $L^{-1}\left\{\frac{1}{p-a}\right\}$

Ans.

$\because L\{e^{at}\} = \frac{1}{p-a}$

$\Rightarrow L^{-1}\left\{\frac{1}{p-a}\right\} = e^{at}$

(4) Find $L^{-1}\left\{\frac{1}{p^2+a^2}\right\}$

Ans:- $\because L\{\sin at\} = \frac{1}{p^2+a^2}$

$\Rightarrow L^{-1}\left\{\frac{1}{p^2+a^2}\right\} = \sin at$

(5) $L^{-1}\left\{\frac{p}{p^2+a^2}\right\}$

Ans: $\because L\{\cos at\} = \frac{p}{p^2+a^2}$

$\Rightarrow L^{-1}\left\{\frac{p}{p^2+a^2}\right\} = \cos at$

(6) $L^{-1}\left\{\frac{p}{p^2-a^2}\right\}$

Ans: $\because L\{\cosh at\} = \frac{p}{p^2-a^2}$

$\Rightarrow L^{-1}\left\{\frac{p}{p^2-a^2}\right\} = \cosh at$

(7) $L^{-1}\left\{\frac{1}{p^2-a^2}\right\}$

Ans: $\because L\{\sinh at\} = \frac{1}{p^2-a^2}$

$\Rightarrow L^{-1}\left\{\frac{1}{p^2-a^2}\right\} = \sinh at$

—x—

(8) $L^{-1}\left\{\frac{1}{p+a}\right\}$

Ans:- $\because L\{e^{-at}\} = \frac{1}{p+a}$

$\Rightarrow L^{-1}\left\{\frac{1}{p+a}\right\} = e^{-at}$ Ans

# Table of inverse Laplace Transform

| No. | $f(p)$ | $L^{-1}\{f(p)\} = F(t)$ |
|---|---|---|
| 1. | $1/p$ | $1$ |
| 2. | $\frac{1}{p^{n+1}}$, $n$ is a +ve integer | $\frac{t^n}{\lfloor n} = \frac{t^n}{\Gamma(n+1)}$ |
| 3. | $\frac{1}{p-a}$ | $e^{at}$ |
| 4. | $\frac{1}{p^2+a^2}$ | $\frac{1}{a}\sin at$ |
| 5. | $\frac{p}{p^2+a^2}$ | $\cos at$ |
| 6. | $\frac{1}{p^2-a^2}$ | $\frac{1}{a}\sinh at$ |
| 7. | $\frac{p}{p^2-a^2}$ | $\cosh at$ |
| 8. | $\frac{1}{p^{n+1}}$, $n > -1$ | $\frac{t^n}{\Gamma(n+1)}$ |

## Problems

(1) Find

(i) $L^{-1}\left\{\frac{1}{p^4}\right\}$

Ans:- $L^{-1}\left\{\frac{1}{p^4}\right\} = \frac{t^{4-1}}{\lfloor 3} = \frac{t^3}{6}$

(ii) $L^{-1}\left\{\frac{1}{p^2+4}\right\}$

Ans:- $L^{-1}\left\{\frac{1}{p^2+4}\right\} = L^{-1}\left\{\frac{1}{p^2+2^2}\right\} = \frac{1}{2}\sin 2t$

(iii) $L^{-1}\left\{\frac{4}{p-2}\right\}$

Ans: $L^{-1}\left\{\frac{4}{p-2}\right\} = 4L^{-1}\left\{\frac{1}{p-2}\right\} = 4e^{2t}$

(iv) $L^{-1}\left\{\frac{1}{\sqrt{p}}\right\}$

Ans: $L^{-1}\left\{\frac{1}{\sqrt{p}}\right\} = L^{-1}\left\{\frac{1}{p^{1/2}}\right\} = \frac{t^{\frac{1}{2}-1}}{\Gamma(1/2)} = \frac{t^{-1/2}}{\sqrt{\pi}} = \frac{1}{\sqrt{\pi t}}$ Ans

—x—

(2) Find

(i) $L^{-1}\left\{\frac{1}{p^{7/2}}\right\}$

Ans: $L^{-1}\left\{\frac{1}{p^{7/2}}\right\} = \frac{t^{\frac{7}{2}-1}}{\Gamma(7/2)} = \frac{t^{5/2}}{\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\Gamma(\frac{1}{2})} = \frac{t^{5/2}}{\frac{15}{8}\sqrt{\pi}} = \frac{8}{15}\frac{t^{5/2}}{\sqrt{\pi}}$

(ii) $L^{-1}\left\{\frac{p}{p^2+2} + \frac{6p}{p^2-16} + \frac{3}{p-3}\right\}$

Ans:

$L^{-1}\left\{\frac{p}{p^2+2} + \frac{6p}{p^2-16} + \frac{3}{p-3}\right\}$

$= L^{-1}\left\{\frac{p}{p^2+2}\right\} + 6L^{-1}\left\{\frac{p}{p^2-16}\right\} + 3L^{-1}\left\{\frac{1}{p-3}\right\}$

$= \cos\sqrt{2}t + 6\cosh 4t + 3e^{3t}$

—x—

(3) Find $L^{-1}\left\{\frac{6}{2p-3} - \frac{3+4p}{9p^2-16} + \frac{8-6p}{16p^2+9}\right\}$

Ans: $L^{-1}\left\{\frac{6}{2p-3}\right\} - L^{-1}\left\{\frac{3+4p}{9p^2-16}\right\} + L^{-1}\left\{\frac{8-6p}{16p^2+9}\right\}$

$= \frac{6}{2}L^{-1}\left\{\frac{1}{p-\frac{3}{2}}\right\} - L^{-1}\left\{\frac{3}{9p^2-16}\right\} - 4L^{-1}\left\{\frac{p}{9p^2-16}\right\} + L^{-1}\left\{\frac{8}{16p^2+9}\right\} - L^{-1}\left\{\frac{6p}{16p^2+}\right.$