$$= 3\,L^{-1}\left\{\frac{1}{p-\frac{3}{2}}\right\} - \frac{3}{9}\,L^{-1}\left\{\frac{1}{p^2-\frac{16}{9}}\right\} - \frac{4}{9}\,L^{-1}\left\{\frac{p}{p^2-16/9}\right\}$$

$$+ \frac{8}{16}\,L^{-1}\left\{\frac{1}{p^2+\frac{9}{16}}\right\} - \frac{6}{16}\,L^{-1}\left\{\frac{p}{p^2+\frac{9}{16}}\right\}$$

$$= 3\,L^{-1}\left\{\frac{1}{p-\frac{3}{2}}\right\} - \frac{1}{3}\,L^{-1}\left\{\frac{1}{p^2-\frac{16}{9}}\right\} - \frac{4}{9}\,L^{-1}\left\{\frac{p}{p^2-\frac{16}{9}}\right\}$$

$$+ \frac{1}{2}\,L^{-1}\left\{\frac{1}{p^2+\frac{9}{16}}\right\} - \frac{3}{8}\,L^{-1}\left\{\frac{p}{p^2+\frac{9}{16}}\right\}$$

$$= 3\,e^{3/2\,t} - \frac{1}{3}\,\frac{1}{4/3}\sinh\frac{4}{3}t - \frac{4}{9}\cosh\frac{4}{3}t$$

$$+ \frac{1}{2}\,\frac{1}{3/4}\sin\frac{3}{4}t - \frac{3}{8}\cos\frac{3}{4}t$$

$$= 3e^{\frac{3}{2}t} - \frac{1}{4}\sinh\frac{4}{3}t - \frac{4}{9}\cosh\frac{4}{3}t + \frac{2}{3}\sin\frac{3}{4}t - \frac{3}{8}\cos\frac{3}{4}t$$

(4) Find $L^{-1}\left\{\frac{3}{p^2-3} + \frac{3p+2}{p^3} - \frac{3p-27}{p^2+9} + \frac{6-30\sqrt{p}}{p^4}\right\}$

Ans:-

$$L^{-1}\left\{\frac{3}{p^2-3} + \frac{3p+2}{p^3} - \frac{3p-27}{p^2+9} + \frac{6-30\sqrt{p}}{p^4}\right\}$$

$$= L^{-1}\left\{\frac{3}{p^2-3}\right\} + 3\,L^{-1}\left\{\frac{1}{p^2}\right\} + 2\,L^{-1}\left\{\frac{1}{p^3}\right\} + 6\,L^{-1}\left\{\frac{1}{p^4}\right\} - 30\,L^{-1}\left\{\frac{\sqrt{p}}{p^4}\right\}$$

$$- L^{-1}\left\{\frac{3p}{p^2+9}\right\} + 27\,L^{-1}\left\{\frac{1}{p^2+9}\right\}$$

$$= 3L^{-1}\left\{\frac{1}{p^2-3}\right\} + 3\,L^{-1}\left\{\frac{1}{p^2}\right\} + 2\,L^{-1}\left\{\frac{1}{p^3}\right\} + 6\,L^{-1}\left\{\frac{1}{p^4}\right\} - 30\,L^{-1}\left\{\frac{1}{p^{7/2}}\right\}$$

$$- 3L^{-1}\left\{\frac{p}{p^2+9}\right\} + 27\,L^{-1}\left\{\frac{1}{p^2+9}\right\}$$

$$= \frac{3}{\sqrt{3}}\sinh\sqrt{3}t + 3\,\frac{t^1}{1!} + 2\,\frac{t^2}{2!} + 6\,\frac{t^3}{3!} - 30\,\frac{t^{\frac{7}{2}-1}}{\Gamma(7/2)} - 3\cos 3t + 9\sin 3t$$

$$= \sqrt{3}\sinh\sqrt{3}t + 3t + t^2 + t^3 - \frac{30\, t^{5/2}}{\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\sqrt{\pi}} - 3\cos 3t + 9\sin 3t$$

$$= \sqrt{3}\sinh\sqrt{3}t + 3t + t^2 + t^3 - \frac{16\, t^{5/2}}{\sqrt{\pi}} - 3\cos 3t + 9\sin 3t$$

—x—

(5) Prove that

$$L^{-1}\left\{\frac{5}{p^2} + \left(\frac{\sqrt{p}-1}{p}\right)^2 - \frac{7}{3p+2}\right\} = 1 + 6t - 4\sqrt{\frac{t}{\pi}} - \frac{7}{3}e^{-2t/3}$$

Soln:

$$L^{-1}\left\{\frac{5}{p^2} + \left(\frac{\sqrt{p}-1}{p}\right)^2 - \frac{7}{3p+2}\right\}$$

$$= L^{-1}\left\{\frac{5}{p^2} + \frac{p - 2\sqrt{p} + 1}{p^2} - \frac{7}{3p+2}\right\}$$

$$= 5L^{-1}\left\{\frac{1}{p^2}\right\} + L^{-1}\left\{\frac{p}{p^2} - \frac{2\sqrt{p}}{p^2} + \frac{1}{p^2}\right\} - L^{-1}\left\{\frac{7}{3p+2}\right\}$$

$$= 5L^{-1}\left\{\frac{1}{p^2}\right\} + L^{-1}\left\{\frac{1}{p}\right\} - 2L^{-1}\left\{\frac{1}{p^{3/2}}\right\} + L^{-1}\left\{\frac{1}{p^2}\right\} - \frac{7}{3}L^{-1}\left\{\frac{1}{p+\frac{2}{3}}\right\}$$

$$= 5\frac{t^1}{\lfloor 1} + 1 - 2\frac{t^{\frac{3}{2}-1}}{\Gamma(3/2)} + \frac{t^1}{\lfloor 1} - \frac{7}{3}e^{-2/3\, t}$$

$$= 5t + 1 - 2\frac{t^{1/2}}{\frac{1}{2}\Gamma(\frac{1}{2})} + t - \frac{7}{3}e^{-2/3\, t}$$

$$= 1 + 6t - 4\frac{\sqrt{t}}{\sqrt{\pi}} - \frac{7}{3}e^{-2/3\, t}$$

$$= 1 + 6t - 4\sqrt{\frac{t}{\pi}} - \frac{7}{3}e^{-2/3\, t}$$ proved

## First translation or shifting theorem

If $L^{-1}\{f(p)\} = F(t)$, then

$$L^{-1}\{f(p-a)\} = e^{at}F(t) = e^{at}L^{-1}\{f(p)\}$$

Ans:

We have, $f(p) = \int_0^\infty e^{-pt}F(t)\,dt$

$$\therefore f(p-a) = \int_0^\infty e^{-(p-a)t}F(t)\,dt$$

$$= \int_0^\infty e^{-pt}\{e^{at}F(t)\}\,dt$$

$$= L\{e^{at}F(t)\}$$

$$\therefore L^{-1}\{f(p-a)\} = e^{at}F(t) = e^{at}L^{-1}\{f(p)\}$$

—x—

## Problems

① Find $L^{-1}\left[\frac{1}{p^2-6p+10}\right]$

Ans:

$$L^{-1}\left[\frac{1}{p^2-6p+10}\right]$$

$$= L^{-1}\left[\frac{1}{p^2-2\cdot p\cdot 3+3^2+10-3^2}\right]$$

$$= L^{-1}\left[\frac{1}{(p-3)^2+1}\right] = e^{3t}L^{-1}\left[\frac{1}{p^2+1}\right]$$

$$= e^{3t}\frac{1}{1}\sin t$$

$= e^{3t}\sin t$ [From 1st shifting theorem]

(2) Find $L^{-1}\left[\frac{1}{p^2+8p+16}\right]$

Ans:

$$L^{-1}\left[\frac{1}{p^2+8p+16}\right]$$

$$= L^{-1}\left[\frac{1}{p^2+2\cdot p\cdot 4+4^2}\right] = L^{-1}\left[\frac{1}{(p+4)^2}\right]$$

$$= e^{-4t}\, L^{-1}\left\{\frac{1}{p^2}\right\}$$

$$= e^{-4t}\frac{t}{1!}$$ [From 1st shifting theorem]

$$= te^{-4t}$$

(3) $L^{-1}\left\{\frac{p-1}{(p+3)(p^2+2p+2)}\right\}$

Ans:-

Let $\frac{p-1}{(p+3)(p^2+2p+2)} = \frac{A}{p+3} + \frac{Bp+C}{p^2+2p+2}$ —— (1)

$\Rightarrow (p-1) = A(p^2+2p+2) + (Bp+C)(p+3)$

$\Rightarrow p-1 = (A+B)p^2 + (2A+3B+C)p + 2A + 3C$

Equating the like co-efficient

$A+B=0$ —— (2)

$2A+3B+C=1$ —— (3)

$2A+3C=-1$ —— (4)

From (2)

$B=-A$

$\therefore$ From (3)

$2A-3A+C=1$

$\Rightarrow -A+C=1$ —— (5)