FOURIER TRANSFORM (INFINITE)

Definition: Let $F(x)$ be a function defined on $(-\infty, \infty)$ and be piecewise continuous in each partial interval and absolutely integrable in $(-\infty, \infty)$. Then the Fourier transform of $F(x)$ is a function of a new variable s and it is denoted and defined as

$$F\{F(x)\} = \bar{F}(s) = f(s) = \int_{-\infty}^{\infty} e^{isx} F(x)\, dx$$

The function $F(x)$ is then called inverse Fourier transform of $\bar{F}(s)$ or $f(s)$ and is denoted by

$$F(x) = F^{-1}\{\bar{F}(s)\} \quad \text{or} \quad F(x) = F^{-1}\{f(s)\}.$$

* Also we may define as: The infinite Fourier transform of $F(x)$ defined in $(-\infty, \infty)$ is denoted by $f(s)$ or $F\{F(x)\}$ and is defined as

$$f(s) = F\{F(x)\} = \int_{-\infty}^{\infty} e^{isx} F(x)\, dx.$$

* **Inversion formula for Fourier transform:**

If $\bar{F}(s)$ is the Fourier transform of $F(x)$ and if $F(x)$ satisfies the Dirichlet conditions in every finite interval $(-l, l)$ and further if $F(x)$ is absolutely integrable in $(-\infty, \infty)$, then at every point of continuity of $F(x)$,

$$F(x) = F^{-1}\{f(s)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(s)\, e^{-isx}\, dx$$

Note: Some authors take the coefficient as $\frac{1}{\sqrt{2\pi}}$ instead of $\frac{1}{2\pi}$.

Definition: Infinite fourier sine transform:

Let $F(x)$ be a function defined on $(-\infty,\infty)$ and be piecewise continuous in each partial interval and absolutely integrable in $(-\infty,\infty)$. Then the infinite fourier sine transform of $f(x)$ is a function of a new variable s and it is denoted and defined as

$$f_s\{F(x)\} = \overline{F}_s(s) = f_s(s) = \int_0^{\infty} F(x) \sin sx \, dx.$$

The function $f(s)$ is then called inverse fourier sine transform of $f_s(s)$ and is denoted by

$$F(x) = F_s^{-1}\{f_s(s)\}.$$

Inversion formula for inverse fourier sine transform

If $f_s(s)$ is the infinite sine transform of $f(x)$ and if $f(x)$ satisfies the Dirichlet conditions in every finite interval $(-l,l)$ and further if $f(x)$ is absolutely integrable in $(-\infty,\infty)$, Then at every point of Continuity of $f(x)$,

$$f(x) = \frac{2}{\pi}\int_0^{\infty} f_s(s) \sin sx \, ds.$$