5) Evaluate:

i) $L^{-1}\left\{\frac{1}{(p+a)^n}\right\}$

Sol:- $L^{-1}\left\{\frac{1}{(p+a)^n}\right\}$

$= e^{-at} L^{-1}\left\{\frac{1}{p^n}\right\}$

$= e^{-at} \frac{t^{n-1}}{\lfloor n-1}$ Ans.

ii) $L^{-1}\left\{\frac{p}{(p+1)^{5/2}}\right\}$

Sol: $\because L^{-1}\left\{\frac{p}{(p+1)^{5/2}}\right\}$

$= L^{-1}\left\{\frac{p+1-1}{(p+1)^{5/2}}\right\} = L^{-1}\left\{\frac{p+1}{(p+1)^{5/2}}\right\} - L^{-1}\left\{\frac{1}{(p+1)^{5/2}}\right\}$

$= L^{-1}\left\{\frac{1}{(p+1)^{3/2}}\right\} - L^{-1}\left\{\frac{1}{(p+1)^{5/2}}\right\}$

$= e^{-t} L^{-1}\left\{\frac{1}{p^{3/2}}\right\} - e^{-t} L^{-1}\left\{\frac{1}{p^{5/2}}\right\}$

$= e^{-t} \frac{t^{\frac{3}{2}-1}}{\Gamma(\frac{3}{2})} - e^{-t} \frac{t^{\frac{5}{2}-1}}{\Gamma(\frac{5}{2})}$

$= e^{-t} \frac{t^{\frac{1}{2}}}{\frac{1}{2}\Gamma(\frac{1}{2})} - e^{-t} \frac{t^{3/2}}{\frac{3}{2}\frac{1}{2}\Gamma(\frac{1}{2})}$

$= 2e^{-t} \frac{\sqrt{t}}{\sqrt{\pi}} - \frac{4}{3} e^{-t} \frac{t^{3/2}}{\sqrt{\pi}}$

$= 2e^{-t} \frac{\sqrt{t}}{\sqrt{\pi}} \left(1 - \frac{2}{3}t\right)$ Ans.

$$\therefore L^{-1}\left\{\frac{p+1}{(p+1)(p^2+2p+2)}\right\} = -\frac{4}{5}e^{-2t} + \frac{4}{5}e^{-t}\cos t - \frac{4}{5}e^{-t}L^{-1}\left\{\frac{p}{p^2+1}\right\}$$
$$+ \frac{1}{5}e^{-t}L^{-1}\left\{\frac{1}{p^2+1}\right\}$$
$$= -\frac{4}{5}e^{-2t} + \frac{4}{5}e^{-t}\cos t - \frac{4}{5}e^{-t}\sin t$$
$$+ \frac{1}{5}e^{-t}\sin t$$
$$= -\frac{4}{5}e^{-2t} + \frac{4}{5}e^{-t}\cos t - \frac{3}{5}e^{-t}\sin t$$
$$= -\frac{4}{5}e^{-2t} + \frac{e^{-t}}{5}(4\cos t - 3\sin t)$$

Ans

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④ Evaluate :

(1) $L^{-1}\left\{\frac{3p-2}{p^2-4p+20}\right\}$

Ans:-

$$\because L^{-1}\left\{\frac{3p-2}{p^2-4p+20}\right\}$$

$$= L^{-1}\left\{\frac{3p}{p^2-4p+20}\right\} - L^{-1}\left\{\frac{2}{p^2-4p+20}\right\}$$

$$= L^{-1}\left\{\frac{3p}{p^2-2\cdot p\cdot 2+2^2+20-2^2}\right\} - L^{-1}\left\{\frac{2}{p^2-2\cdot p\cdot 2+2^2+20-2^2}\right\}$$

$$= 3L^{-1}\left\{\frac{p}{(p-2)^2+16}\right\} - 2L^{-1}\left\{\frac{1}{(p-2)^2+16}\right\}$$

$$= 3L^{-1}\left\{\frac{p-2+2}{(p-2)^2+16}\right\} - 2L^{-1}\left\{\frac{1}{(p-2)^2+16}\right\}$$

$$= 3L^{-1}\left\{\frac{p-2}{(p-2)^2+16}\right\} + 6L^{-1}\left\{\frac{1}{(p-2)^2+16}\right\} - 2L^{-1}\left\{\frac{1}{(p-2)^2+16}\right\}$$

$$= 3e^{2t}L^{-1}\left\{\frac{p}{p^2+16}\right\} + 6e^{2t}L^{-1}\left\{\frac{1}{p^2+16}\right\} - 2e^{2t}L^{-1}\left\{\frac{1}{p^2+}\right.$$

Operating (5)×2 + (4) we get

$$5C = 1$$
$$\Rightarrow C = \frac{1}{5}$$

From (5)

$$A = \frac{1}{5} - 1 = -\frac{4}{5}$$

From (2) $B = -\left(-\frac{4}{5}\right) = \frac{4}{5}$

$\therefore$ From (1)

$$\frac{p-1}{(p+3)(p^2+2p+2)} = \frac{-4/5}{p+3} + \frac{\frac{4}{5}p+\frac{1}{5}}{p^2+2p+2}$$

$$= -\frac{4}{5}\frac{1}{(p+3)} + \frac{1}{5}\cdot\frac{4p+1}{p^2+2p+2}$$

$$\therefore L^{-1}\left\{\frac{p-1}{(p+3)(p^2+2p+2)}\right\} = L^{-1}\left\{-\frac{4}{5}\frac{1}{(p+3)} + \frac{1}{5}\frac{4p+1}{p^2+2p+2}\right\}$$

$$= L^{-1}\left\{-\frac{4}{5}\frac{1}{(p+3)} + \frac{4}{5}\frac{p}{p^2+2p+2} + \frac{1}{5}\frac{1}{p^2+2p+2}\right\}$$

$$= L^{-1}\left\{-\frac{4}{5}\frac{1}{(p+3)}\right\} + \frac{4}{5}L^{-1}\left\{\frac{p}{p^2+2\cdot p\cdot 1+1^2+2-1^2}\right\}$$
$$+ \frac{1}{5}L^{-1}\left\{\frac{1}{p^2+2\cdot p\cdot 1+1^2+2-1^2}\right\}$$

$$= -\frac{4}{5}L^{-1}\left\{\frac{1}{p+3}\right\} + \frac{4}{5}L^{-1}\left\{\frac{p}{(p^2+2p+1)+1}\right\} + \frac{1}{5}L^{-1}\left\{\frac{1}{(p^2+2p+1)+1}\right\}$$

$$= -\frac{4}{5}L^{-1}\left\{\frac{1}{p+3}\right\} + \frac{4}{5}L^{-1}\left\{\frac{p+1-1}{(p+1)^2+1}\right\} + \frac{1}{5}L^{-1}\left\{\frac{1}{(p+1)^2+1}\right\}$$

$$= -\frac{4}{5}e^{-3t} + \frac{4}{5}e^{-t}L^{-1}\left\{\frac{p}{p^2+1}\right\} - \frac{4}{5}L^{-1}\left\{\frac{1}{(p+1)^2+1}\right\} + \frac{1}{5}L^{-1}\left\{\frac{1}{(p+1)^2}\right.$$

$$\therefore L^{-1}\left\{\frac{3p-2}{p^2-4p+20}\right\} = 3e^{2t}\cos 4t + 6e^{2t}\frac{\sin 4t}{4} - 2e^{2t}\frac{\sin 4t}{4}$$
$$= 3e^{2t}\cos 4t + e^{2t}\sin 4t$$

—x—

(ii) $L^{-1}\left\{\frac{3p+7}{p^2-2p-3}\right\}$

Ans.

$$\because L^{-1}\left\{\frac{3p+7}{p^2-2p-3}\right\}$$
$$= L^{-1}\left\{\frac{3p+7}{p^2-2\cdot p\cdot 1+1^2-3-1^2}\right\}$$
$$= L^{-1}\left\{\frac{3p+7}{(p-1)^2-4}\right\} = L^{-1}\left\{\frac{3p}{(p-1)^2-4}\right\} + L^{-1}\left\{\frac{7}{(p-1)^2-4}\right\}$$
$$= 3L^{-1}\left\{\frac{p}{(p-1)^2-4}\right\} + 7L^{-1}\left\{\frac{1}{(p-1)^2-4}\right\}$$
$$= 3L^{-1}\left\{\frac{(p-1)+1}{(p-1)^2-4}\right\} + 7L^{-1}\left\{\frac{1}{(p-1)^2-4}\right\}$$
$$= 3L^{-1}\left\{\frac{p-1}{(p-1)^2-4}\right\} + 3L^{-1}\left\{\frac{1}{(p-1)^2-4}\right\} + 7L^{-1}\left\{\frac{1}{(p-1)^2-4}\right\}$$
$$= 3e^{t}L^{-1}\left\{\frac{p}{p^2-4}\right\} + 3e^{t}L^{-1}\left\{\frac{1}{p^2-4}\right\} + 7e^{t}L^{-1}\left\{\frac{1}{p^2-4}\right\}$$
$$= 3e^{t}\cosh 2t + 3e^{t}\frac{\sinh 2t}{2} + 7e^{t}\frac{\sinh 2t}{2}$$
$$= 3e^{t}\cosh 2t + \frac{10}{2}e^{t}\sinh 2t$$
$$= 3e^{t}\left(\frac{e^{2t}+e^{-2t}}{2}\right) + 5e^{t}\left(\frac{e^{2t}-e^{-2t}}{2}\right)$$
$$= \frac{3}{2}e^{3t} + \frac{3}{2}e^{-t} + \frac{5}{2}e^{3t} - \frac{5}{2}e^{-t}$$
$$= 4e^{3t} - e^{-t}$$ Ans.