(5) Evaluate:

(i) $L^{-1}\left\{\frac{1}{(p+a)^n}\right\}$

Ans:- $L^{-1}\left\{\frac{1}{(p+a)^n}\right\}$

$= e^{-at} L^{-1}\left\{\frac{1}{p^n}\right\}$

$= e^{-at} \frac{t^{n-1}}{\lfloor n-1}$ Ans

(ii) $L^{-1}\left\{\frac{p}{(p+1)^{5/2}}\right\}$

Ans:

$\because L^{-1}\left\{\frac{p}{(p+1)^{5/2}}\right\}$

$= L^{-1}\left\{\frac{p+1-1}{(p+1)^{5/2}}\right\} = L^{-1}\left\{\frac{p+1}{(p+1)^{5/2}}\right\} - L^{-1}\left\{\frac{1}{(p+1)^{5/2}}\right\}$

$= L^{-1}\left\{\frac{1}{(p+1)^{3/2}}\right\} - L^{-1}\left\{\frac{1}{(p+1)^{5/2}}\right\}$

$= e^{-t} L^{-1}\left\{\frac{1}{p^{3/2}}\right\} - e^{-t} L^{-1}\left\{\frac{1}{p^{5/2}}\right\}$

$= e^{-t} \frac{t^{\frac{3}{2}-1}}{\Gamma(\frac{3}{2})} - e^{-t} \frac{t^{\frac{5}{2}-1}}{\Gamma(\frac{5}{2})}$

$= e^{-t} \frac{t^{\frac{1}{2}}}{\frac{1}{2}\Gamma(\frac{1}{2})} - e^{-t} \frac{t^{3/2}}{\frac{3}{2}\frac{1}{2}\Gamma(\frac{1}{2})}$

$= 2e^{-t} \frac{\sqrt{t}}{\sqrt{\pi}} - \frac{4}{3} e^{-t} \frac{t^{3/2}}{\sqrt{\pi}}$

$= 2e^{-t} \frac{\sqrt{t}}{\sqrt{\pi}} \left(1 - \frac{2}{3} t\right)$ Ans

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(iii) $L^{-1}\left\{\frac{p}{(p+1)^5}\right\}$

Ans:-

$$L^{-1}\left\{\frac{p}{(p+1)^5}\right\}$$

$$= L^{-1}\left\{\frac{(p+1)-1}{(p+1)^5}\right\} = L^{-1}\left\{\frac{1}{(p+1)^4}\right\} - L^{-1}\left\{\frac{1}{(p+1)^5}\right\}$$

$$= e^{-t} L^{-1}\left\{\frac{1}{p^4}\right\} - e^{-t} L^{-1}\left\{\frac{1}{p^5}\right\}$$

$$= e^{-t} \frac{t^{4-1}}{\lfloor 4-1} - e^{-t} \frac{t^{5-1}}{\lfloor 5-1}$$

$$= e^{-t} \frac{t^3}{\lfloor 3} - e^{-t} \frac{t^4}{\lfloor 4}$$

$$= \frac{e^{-t} t^3}{6}\left(1 - \frac{t}{4}\right) \text{ Ans.}$$

—x—

(iv) $L^{-1}\left\{\frac{3p+2}{4p^2+12p+9}\right\}$

Ans:-

$$L^{-1}\left\{\frac{3p+2}{4p^2+12p+9}\right\}$$

$$= \frac{1}{4} L^{-1}\left\{\frac{3p+2}{p^2+3p+\frac{9}{4}}\right\}$$

$$= \frac{1}{4} L^{-1}\left\{\frac{3p+2}{p^2+2\cdot p\cdot\frac{3}{2}+\left(\frac{3}{2}\right)^2}\right\}$$

$$= \frac{1}{4} L^{-1}\left\{\frac{3p+2}{\left(p+\frac{3}{2}\right)^2}\right\}$$

$$= \frac{3}{4} L^{-1}\left\{\frac{p}{\left(p+\frac{3}{2}\right)^2}\right\} + \frac{1}{2} L^{-1}\left\{\frac{1}{\left(p+\frac{3}{2}\right)^2}\right\}$$

$$\therefore L^{-1}\left\{\frac{3p+2}{4p^2+12p+9}\right\} = \frac{3}{4}L^{-1}\left\{\frac{(p+\frac{3}{2})-\frac{3}{2}}{(p+\frac{3}{2})^2}\right\} + \frac{1}{2}L^{-1}\left\{\frac{1}{(p+\frac{3}{2})^2}\right\}$$

$$= \frac{3}{4}L^{-1}\left\{\frac{1}{(p+\frac{3}{2})}\right\} - \frac{9}{8}L^{-1}\left\{\frac{1}{(p+\frac{3}{2})^2}\right\} + \frac{1}{2}L^{-1}\left\{\frac{1}{(p+\frac{3}{2})^2}\right\}$$

$$= \frac{3}{4}e^{-3/2t} - \frac{9}{8}e^{-\frac{3}{2}t}L^{-1}\left\{\frac{1}{p^2}\right\} + \frac{1}{2}e^{-3/2t}L^{-1}\left\{\frac{1}{p^2}\right\}$$

$$= \frac{3}{4}e^{-\frac{3}{2}t} - \frac{9}{8}e^{-\frac{3}{2}t}\cdot\frac{t}{\lfloor 1} + \frac{1}{2}e^{-\frac{3}{2}t}\frac{t}{\lfloor 1}$$

$$= \frac{3}{4}e^{-\frac{3}{2}t} - \frac{5}{8}te^{-\frac{3}{2}t}$$

$$= \frac{1}{8}\left(6e^{-\frac{3}{2}t} - 5te^{-\frac{3}{2}t}\right) \text{ Ans.}$$

—x—

(6) Evaluate : $L^{-1}\left\{\frac{p+1}{p^2+6p+25}\right\}$

Ans:-

$$L^{-1}\left\{\frac{p+1}{p^2+6p+25}\right\}$$

$$= L^{-1}\left\{\frac{p+1}{p^2+2\cdot p\cdot 3+3^2+25-3^2}\right\} = L^{-1}\left\{\frac{p+1}{(p+3)^2+16}\right\}$$

$$= L^{-1}\left\{\frac{(p+3)-2}{(p+3)^2+16}\right\} = L^{-1}\left\{\frac{(p+3)}{(p+3)^2+16}\right\} - 2L^{-1}\left\{\frac{1}{(p+3)^2+16}\right\}$$

$$= e^{-3t}L^{-1}\left\{\frac{p}{p^2+16}\right\} - 2e^{-3t}L^{-1}\left\{\frac{1}{p^2+16}\right\}$$

$$= e^{-3t}\cos 4t - 2e^{-3t}\frac{\sin 4t}{4}$$

$$= e^{-3t}\left[\cos 4t - \frac{1}{2}\sin 4t\right] \text{ Ans.}$$

(7) Evaluate: $L^{-1}\left\{\frac{6p^2+22p+18}{p^3+6p^2+11p+6}\right\}$

Ans:- $L^{-1}\left\{\frac{6p^2+22p+18}{p^3+6p^2+11p+6}\right\}$ ——— (1)

Let $f(p) = p^3+6p^2+11p+6$

$\because$ $p=-1$ satisfied $f(p)$

$$\therefore\ p+1)\ p^3+6p^2+11p+6\ (\ p^2+5p+6$$
$$\underline{-p^3 \pm p^2}$$
$$5p^2+11p+6$$
$$\underline{-5p^2 \pm 5p}$$
$$6p+6$$
$$\underline{-6p \pm 6}$$
$$0$$

$$\therefore f(p) = (p+1)(p^2+5p+6)$$
$$= (p+1)(p^2+2p+3p+6)$$
$$= (p+1)\{p(p+2)+3(p+2)\}$$
$$= (p+1)(p+2)(p+3)$$

$\therefore$ (1) becomes

$$L^{-1}\left\{\frac{6p^2+22p+18}{p^3+6p^2+11p+6}\right\} = L^{-1}\left\{\frac{6p^2+22p+18}{(p+1)(p+2)(p+3)}\right\} \quad \text{—— (2)}$$

Let $\frac{6p^2+22p+18}{(p+1)(p+2)(p+3)} = \frac{A}{(p+1)} + \frac{B}{(p+2)} + \frac{C}{(p+3)}$ ——— (3)

$\Rightarrow 6p^2+22p+18 = A(p+2)(p+3)+B(p+1)(p+3)+C(p+1$