APPLICATIONS

PROBLEM:→ Q1: Set up the Langrangean for one dimensional harmonic oscillator and obtain the equation describing its motion.

ANSWER:→ Let us consider the motion of a Linear harmonic oscillator, of mass "m" along x direction. For the displacement x of the oscillator measured from its mean position, we will have,

(i) The Kinetic energy of this oscillator

$$T = \frac{1}{2} m\dot{x}^2$$

(ii) The Potential energy of this oscillator

$$V = -\int F\,dx$$

$$= -\int (-Kx)\,dx$$ ---- Where K being a Proportionality constant.

$$= \frac{Kx^2}{2}$$

$$= \frac{1}{2} Kx^2$$

Hence The Langrangean of this System

$$L = (T - V)$$

$$= \frac{1}{2} m\dot{x}^2 - \frac{1}{2} Kx^2$$

This gives, $\frac{\partial L}{\partial \dot{x}} = m\dot{x}$

or, $\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = m\ddot{x}$

And, $\frac{\partial L}{\partial x} = -Kx$

Now, using Langranges equation,

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0$$

$$\Rightarrow \boxed{m\ddot{x} + Kx = 0}$$

Which is the required equation. For Linear harmonic oscillators.

[2.] Motion of a Particle under a force :→ (I)

ANS:→ Let us consider a particle mass "m" moving under a central force ($F = -\frac{K}{r^2}$). At a point having polar co-ordinate (r, θ), well have the kinetic energy of particle,

$$T = \frac{1}{2} m \left[\dot{r}^2 + (r\dot{\theta})^2 \right]$$

And potential energy of particle,

$$V = -\int F\,dr$$
$$= -\int -\frac{K}{r^2}\,dr,$$
$$= -\frac{K}{r}$$

∴ Langrangian of the particle

$$L = T - V$$
$$= \frac{1}{2} m \left[\dot{r}^2 + (r\dot{\theta})^2 \right] + \frac{K}{r}$$

This equation gives,

[1] For r- co-ordinate :→

$$\frac{\partial L}{\partial \dot{r}} = m\dot{r}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{r}} = m\ddot{r}$$

and, $$\frac{\partial L}{\partial r} = mr\dot{\theta}^2 - \frac{K}{r^2}$$

Hence the Langrange's equation

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{r}} - \frac{\partial L}{\partial r} = 0$$

⇒

$$\boxed{m\ddot{r} - mr\dot{\theta}^2 + \frac{K}{r^2} = 0}$$

ii) For θ- Co-ordinate :→

$$\frac{\partial L}{\partial \dot{\theta}} = mr^2\dot{\theta}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = \frac{d}{dt}(mr^2\dot{\theta})$$

and $\frac{\partial L}{\partial \theta} = 0$,

Hence, The Langrang's Equation gives,

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = 0$$

$$\Rightarrow \boxed{\frac{d}{dt}(mr^2\dot{\theta}) = 0} \quad (2)$$

Equation (1) & (2) are the required Equations.