Putting $p = -1$ in (4)

$$6 - 22 + 18 = A \times 1 \times 2 + 0 + 0$$

$$\Rightarrow A = \frac{2}{2} = 1$$

Putting $p = -2$ in (4)

$$6 \times 4 - 22 \times 2 + 18 = 0 + B \times (-1) \times (1)$$

$$\Rightarrow 24 - 44 + 18 = -B$$

$$\Rightarrow B = 2$$

Putting $p = -3$ in (4)

$$6 \times 9 - 22 \times 3 + 18 = 0 + 0 + C \times (-2) \times (-1)$$

$$\Rightarrow 54 - 66 + 18 = 2C$$

$$\Rightarrow C = \frac{6}{2} = 3$$

$\therefore$ From (2) and (3)

$$L^{-1}\left\{\frac{6p^2 + 22p + 18}{p^3 + 6p^2 + 11p + 6}\right\} = L^{-1}\left\{\frac{1}{p+1} + \frac{2}{p+2} + \frac{3}{p+3}\right\}$$

$$= L^{-1}\left\{\frac{1}{p+1}\right\} + L^{-1}\left\{\frac{2}{p+2}\right\} + L^{-1}\left\{\frac{3}{p+3}\right\}$$

$$= L^{-1}\left\{\frac{1}{(p+1)}\right\} + 2L^{-1}\left\{\frac{1}{(p+2)}\right\} + 3L^{-1}\left\{\frac{1}{p+3}\right\}$$

$$= e^{-t} + 2e^{-2t} + 3e^{-3t} \quad \text{Ans}$$

—x—

(8) Prove that

$$L^{-1}\left\{\frac{4p+5}{(p-1)^2(p+2)}\right\} = 3te^{t} + \frac{1}{3}e^{t} - \frac{1}{3}e^{-2t}$$

Sol:

$$\text{L.H.S} = L^{-1}\left\{\frac{4p+5}{(p-1)^2(p+2)}\right\} \quad \text{—— (1)}$$

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Let $\frac{4p+5}{(p-1)^2(p+2)} = \frac{A}{p-1} + \frac{B}{(p-1)^2} + \frac{C}{(p+2)}$ —— (2)

$\Rightarrow \quad 4p+5 = A(p-1)(p+2) + B(p+2) + C(p-1)^2$

$\Rightarrow \quad 4p+5 = A(p^2+p-2) + B(p+2) + C(p^2-2p+1)$

$\Rightarrow \quad 4p+5 = (A+C)p^2 + (A+B-2C)p - 2A+2B+C$

Equating the like co-efficient

$A+C=0$ —— (3)

$A+B-2C = 4$ —— (4)

$-2A+2B+C = 5$ —— (5)

From (3),

$A = -C$

From (4) and (5)

$-C+B-2C=4$

and $+2C+2B+C=5$

$\Rightarrow \quad B-3C = 4$ —— (6)

and $2B+3C=5$ —— (7)

Adding both (6) and (7)

$3B = 9$

$\Rightarrow B = 3$

From (6)

$3C = B-4 = 3-4$

$\Rightarrow C = -\frac{1}{3}$

$\therefore$ From (3)

$A = \frac{1}{3}$

$\therefore$ From (1) and (2)

$$L^{-1}\left\{\frac{4p+5}{(p-1)^2(p+2)}\right\} = L^{-1}\left\{\frac{\frac{1}{3}}{p-1} + \frac{3}{(p-1)^2} + \frac{-\frac{1}{3}}{(p+2)}\right\}$$

$$\therefore L^{-1}\left\{\frac{4p+5}{(p-1)^2(p+2)}\right\} = \frac{1}{3}L^{-1}\left\{\frac{1}{p-1}\right\} + 3L^{-1}\left\{\frac{1}{(p-1)^2}\right\} - \frac{1}{3}L^{-1}\left\{\frac{1}{p+2}\right\}$$

$$= \frac{1}{3}e^{t} + 3e^{t}L^{-1}\left\{\frac{1}{p^2}\right\} - \frac{1}{3}e^{-2t}$$

$$= \frac{1}{3}e^{t} + 3e^{t}\frac{t}{\underline{1}} - \frac{1}{3}e^{-2t}$$

$$= \frac{1}{3}e^{t} + 3te^{t} - \frac{1}{3}e^{-2t}$$

—x—

(9) Find $L^{-1}\left\{\frac{1}{(p+1)(p^2+1)}\right\}$

Ans:

$$L^{-1}\left\{\frac{1}{(p+1)(p^2+1)}\right\} \quad \text{—— (1)}$$

Let $\frac{1}{(p+1)(p^2+1)} = \frac{A}{p+1} + \frac{Bp+C}{p^2+1}$ —— (2)

$\Rightarrow 1 = A(p^2+1) + (Bp+C)(p+1)$

$\Rightarrow 1 = Ap^2 + A + Bp^2 + Bp + Cp + C$

$\Rightarrow 1 = (A+B)p^2 + (B+C)p + A + C$

Equating the like co-efficient

$A + B = 0$ —— (3)

$B + C = 0$ —— (4)

$A + C = 1$ —— (5)

From (3) and (4)

$A - C = 0$ —— (6)

Adding (5) and (6)

$2A = 1 \Rightarrow A = \frac{1}{2}$

$\therefore B = -A = -\frac{1}{2}$ {From (3)}

$\therefore C = A = \frac{1}{2}$ {From (6)}

$\therefore$ From (1) and (2)

$$L^{-1}\left\{\frac{1}{(p+1)(p^2+1)}\right\} = L^{-1}\left\{\frac{1/2}{p+1} + \frac{-\frac{1}{2}p+\frac{1}{2}}{p^2+1}\right\}$$

$$= \frac{1}{2}L^{-1}\left\{\frac{1}{p+1}\right\} - \frac{1}{2}L^{-1}\left\{\frac{p-1}{p^2+1}\right\}$$

$$= \frac{1}{2}e^{-t} - \frac{1}{2}L^{-1}\left\{\frac{p}{(p^2+1)}\right\} + \frac{1}{2}L^{-1}\left\{\frac{1}{p^2+1}\right\}$$

$$= \frac{1}{2}e^{-t} - \frac{1}{2}\cos t + \frac{1}{2}\sin t$$

$$= \frac{1}{2}\left[e^{-t} - \cos t + \sin t\right] \quad \text{Ans}$$

—x—

Date 15/12/2018

*(10) If $L^{-1}\left\{\frac{e^{-1/p}}{p^{1/2}}\right\} = \frac{\cos 2\sqrt{t}}{\sqrt{\pi t}}$, find $L^{-1}\left\{\frac{e^{-a/p}}{p^{1/2}}\right\}$

where $a > 0$

Ans:

Since $L^{-1}\left\{\frac{e^{-1/p}}{p^{1/2}}\right\} = \frac{\cos 2\sqrt{t}}{\sqrt{\pi t}}$ (given)

$$\therefore L^{-1}\left\{\frac{e^{-1/pk}}{(pk)^{1/2}}\right\} = \frac{1}{k}\,\frac{\cos 2\sqrt{t/k}}{\sqrt{\pi \frac{t}{k}}}$$ {By change scale prop

$$\text{or, } L^{-1}\left\{\frac{e^{-1/pk}}{p^{1/2}}\right\} = \frac{\cos 2\sqrt{t/k}}{\sqrt{k}\sqrt{\pi t}}$$

Taking $k = \frac{1}{a}$. we have

$$L^{-1}\left\{\frac{e^{-a/p}}{p^{1/2}}\right\} = \sqrt{a}\,\frac{\cos 2\sqrt{at}}{\sqrt{\pi t}}$$

—x—