$$\Rightarrow \frac{p}{(p^2+1)^2} = -\frac{1}{2}\frac{d}{dp}\left(\frac{1}{p^2+1}\right)$$

$$\therefore L^{-1}\left\{\frac{p}{(p^2+1)^2}\right\} = L^{-1}\left\{-\frac{1}{2}\frac{d}{dp}\left(\frac{1}{p^2+1}\right)\right\}$$

$$= -\frac{1}{2}L^{-1}\left\{\frac{d}{dp}\left(\frac{1}{p^2+1}\right)\right\}$$

$$= -\frac{1}{2}(-1)t\, L^{-1}\left\{\frac{1}{p^2+1}\right\}$$

$$\therefore L^{-1}\left\{\frac{p}{(p^2+1)^2}\right\} = \frac{t}{2}\sin t \quad \text{— (2)}$$

$\therefore$ From (2) and (3)

$$L^{-1}\left\{\frac{p+1}{(p^2+2p+2)^2}\right\} = e^{-t}\cdot\frac{t}{2}\sin t$$

$$= \frac{1}{2}e^{-t}\cdot t\cdot \sin t \quad \text{Ans}$$

—x—

VVVVI

(5) Find $L^{-1}\left\{\log\left(1+\frac{1}{p^2}\right)\right\}$

Ans:

Let $f(p) = \log\left(1+\frac{1}{p^2}\right)$

$$= \log\left(\frac{p^2+1}{p^2}\right)$$

$$= \log(p^2+1) - \log p^2$$

$$\therefore f'(p) = \frac{1}{p^2+1}2p - \frac{1}{p^2}2p$$

$$= 2\left(\frac{p}{p^2+1} - \frac{1}{p}\right)$$

$$\therefore L^{-1}\{f'(p)\} = L^{-1}\left\{2\left(\frac{p}{p^2+1} - \frac{1}{p}\right)\right\}$$

$$\Rightarrow (-1)t\, L^{-1}\{f(p)\} = 2L^{-1}\left\{\frac{p}{p^2+1}\right\} - 2L^{-1}\left\{\frac{1}{p}\right\}$$

vvvs

③ Find $L^{-1}\left\{\frac{p}{(p^2+a^2)^2}\right\}$

Ans:

Since $\frac{d}{dp}\left(\frac{1}{p^2+a^2}\right) = -\frac{2p}{(p^2+a^2)^2}$

$\therefore \frac{p}{(p^2+a^2)^2} = -\frac{1}{2}\frac{d}{dp}\left(\frac{1}{p^2+a^2}\right)$

$\therefore L^{-1}\left\{\frac{p}{(p^2+a^2)^2}\right\} = L^{-1}\left\{-\frac{1}{2}\frac{d}{dp}\left(\frac{1}{p^2+a^2}\right)\right\}$

$= -\frac{1}{2}L^{-1}\left\{\frac{d}{dp}\left(\frac{1}{p^2+a^2}\right)\right\}$

$= -\frac{1}{2}(-1)t\, L^{-1}\left\{\frac{1}{p^2+a^2}\right\}$

$= \frac{1}{2}t \times \frac{1}{a}\sin at$

$= \frac{1}{2a}t\sin at$ Ans

vvvs

④ Find $L^{-1}\left\{\frac{p+1}{(p^2+2p+2)^2}\right\}$

Ans:- We have,

$L^{-1}\left\{\frac{p+1}{(p^2+2p+2)^2}\right\}$

$= L^{-1}\left\{\frac{p+1}{(p^2+2\cdot p\cdot 1+1^2+2-1)^2}\right\}$

$= L^{-1}\left\{\frac{p+1}{\{(p+1)^2+1\}^2}\right\}$

$= e^{-t}L^{-1}\left\{\frac{p}{(p^2+1)^2}\right\}$ —— ①

$\because \frac{d}{dp}\left(\frac{1}{p^2+1}\right) = -\frac{1}{(p^2+1)^2}2p$

② Find (i) $L^{-1}\left\{\frac{1}{(p+a)^3}\right\}$ (ii) $L^{-1}\left\{\frac{1}{(p+a)^n}\right\}$

(i) since $\frac{d^2}{dp^2}\left[\frac{1}{p+a}\right] = \frac{2}{(p+a)^3}$

$\therefore \frac{1}{(p+a)^3} = \frac{1}{2}\frac{d^2}{dp^2}\left(\frac{1}{p+a}\right)$

$\therefore L^{-1}\left\{\frac{1}{(p+a)^3}\right\} = L^{-1}\left\{\frac{1}{2}\frac{d^2}{dp^2}\left(\frac{1}{p+a}\right)\right\}$

$= \frac{1}{2}L^{-1}\left\{\frac{d^2}{dp^2}\left(\frac{1}{p+a}\right)\right\}$

$= \frac{1}{2}(-1)^2 t^2 \, L^{-1}\left\{\frac{1}{p+a}\right\}$

$= \frac{1}{2}t^2 e^{-at}$ Ans

(ii) since $\frac{d^{n-1}}{dp^{n-1}}\left[\frac{1}{p+a}\right] = \frac{(-1)^{n-1}\lfloor n-1}{(p+a)^n}$

$\therefore \frac{1}{(p+a)^n} = \frac{1}{(-1)^{n-1}\lfloor n-1}\frac{d^{n-1}}{dp^{n-1}}\left[\frac{1}{p+a}\right]$

$\therefore L^{-1}\left\{\frac{1}{(p+a)^n}\right\} = L^{-1}\left\{\frac{1}{(-1)^{n-1}\lfloor n-1}\frac{d^{n-1}}{dp^{n-1}}\left[\frac{1}{p+a}\right]\right\}$

$= \frac{1}{(-1)^{n-1}\lfloor n-1} L^{-1}\left\{\frac{d^{n-1}}{dp^{n-1}}\left(\frac{1}{p+a}\right)\right\}$

$= \frac{1}{(-1)^{n-1}\lfloor n-1}(-1)^{n-1}t^{n-1}L^{-1}\left(\frac{1}{p+a}\right)$

$= \frac{t^{n-1}}{\lfloor n-1} e^{-at}$

Ans