Solved Problems

Ex Solve $(D^2+2DD'+D'^2)z = 2\cos y - x\sin y$.

Solⁿ Auxiliary equation is

$$m^2+2m+1=0$$
$$\Rightarrow (m+1)^2=0$$
$$\Rightarrow m=-1,-1$$

So $CF = f_1(y-x) + x f_2(y-x)$

Now, $PI = \dfrac{1}{D^2+2DD'+D'^2}\cdot(2\cos y - x\sin y)$

$$= \frac{1}{(D+D')^2}(2\cos y - x\sin y)$$

$$= \frac{1}{D+D'}\cdot\frac{1}{D+D'}(2\cos y - x\sin y)$$

$$= \frac{1}{D+D'}\int\{2\cos(a+x) - x\sin(a+x)\}dx$$

$$= \frac{1}{D+D'}\left[2\sin(a+x) - \left\{-x\cos(a+x) + \int \cos(x+a)\,dx\right\}\right]$$

$$= \frac{1}{D+D'}\left[2\sin(a+x) + x\cos(a+x) - \sin(a+x)\right]$$

$$= \frac{1}{D+D'}\left[\sin(a+x) + x\cos(a+x)\right]$$

$$= \frac{1}{D+D'}(\sin y + x\cos y) \quad (\text{Putting } a = y-x)$$

$$= \int \left\{ \sin(b+x) + x\cos(b+x) \right\} dx$$

$$= -\cos(b+x) + \left\{ x\sin(b+x) - \int \sin(b+x)\, dx \right\}$$

$$= \cancel{-\cos(b+x)} + x\sin(b+x) + \cancel{\cos(b+x)}$$

$$= x\sin(b+x)$$

$$= x\sin y \quad (\text{Putting } b = y-x)$$

Hence complete solution is

$$z = f_1(y-x) + x f_2(y-x) + x\sin y \quad \text{Ans}$$

__Ex__ Solve: $(D^3 - 4D^2D' + 4DD'^2)z = \cos(2x+y)$

__Sol^n__ Auxiliary eqn is

$$m^3 - 4m^2 + 4m = 0$$

$$\Rightarrow m(m^2 - 4m + 4) = 0$$

$$\Rightarrow m.(m-2) = 0$$

$$\Rightarrow m = 0, 2, 2$$

Hence, $CF = f_1(y) + f_2(y+2x) + x f_3(y+2x)$

Now, $$PI = \frac{1}{D^3 - 4D^2D' + 4DD'^2} \cos(2x+y)$$

$$= \frac{1}{D(D-2D')^2} \cos(2x+y)$$

$$= \frac{1}{(D-2D')^2} \cdot \frac{1}{D} \sin(2x+y)$$

$$= \frac{1}{(D-2D')^2} \frac{\{-\cos(2x+y)\}}{2}$$

$$= -\frac{1}{2} \frac{1}{D^2-2D'} \cos(2x+y)$$

$$= -\frac{1}{2} \frac{x^2}{1^2 \cdot \lfloor 2} \cos(2x+y).$$

Hence complete solution is

$$z = f_1(y) + f_2(y+2x) + x f_3(y+2x) - \frac{1}{4} x^2 \cos(2x+y)$$

Ex 3 Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y \sin x.$

Solⁿ Given P.D.E can be written as

$$(D^2 + DD' - 6D'^2) z = y \sin x$$

So auxiliary equation is

$$m^2 + m - 6 = 0$$

$$\Rightarrow m^2 + 3m - 2m - 6 = 0$$

$$\Rightarrow m(m+3) - 2(m+3) = 0$$

$$\Rightarrow (m+3)(m-2) = 0$$

$$\Rightarrow m = 2, -3.$$

So $CF = f_1(y+2x) + f_2(y-3x)$.

Now. $PI = \dfrac{1}{D^2 + DD' - 6D'^2}\, y \sin x$

$$= \frac{1}{(D-2D')(D+3D')}\, y \sin x$$

$$= \frac{1}{D-2D'} \int (a+3x) \sin x \, dx$$

$$= \frac{1}{D-2D'} \left[(a+3x)(-\cos x) - \int 3(-\cos x)\, dx \right]$$

$$= \frac{1}{D-2D'} \left[-(a+3x)\cos x + 3\sin x \right]$$

$$= \frac{1}{D-2D'} \left[-y\cos x + 3\sin x \right]$$

$$= \int \left[-(b-2x)\cos x + 3\sin x \right] dx$$

$$= -\left[(b-2x)\sin x - \int -2 \cdot \sin x \, dx \right] + 3(-\cos x)$$

$$= -\left[(b-2x)\sin x + 2(-\cos x) \right] - 3\cos x$$

$$= -(b-2x)\sin x + 2\cos x - 3\cos x$$

$$= -y\sin x - \cos x \quad (\text{Put } b = y+2x)$$

Hence complete solⁿ is

$$y = f_1(y+2x) + f_2(y-3x) - y\sin x - \cos x$$

Ans