EXPRESSION FOR ANGULAR VELOCITY COMPONENT

From figure ①, ② & ③, it is clear that the angular velocity $\dot{\phi}$, $\dot{\theta}$ and $\dot{\psi}$ are directed along OZ, OK and OZ' direction respectively.

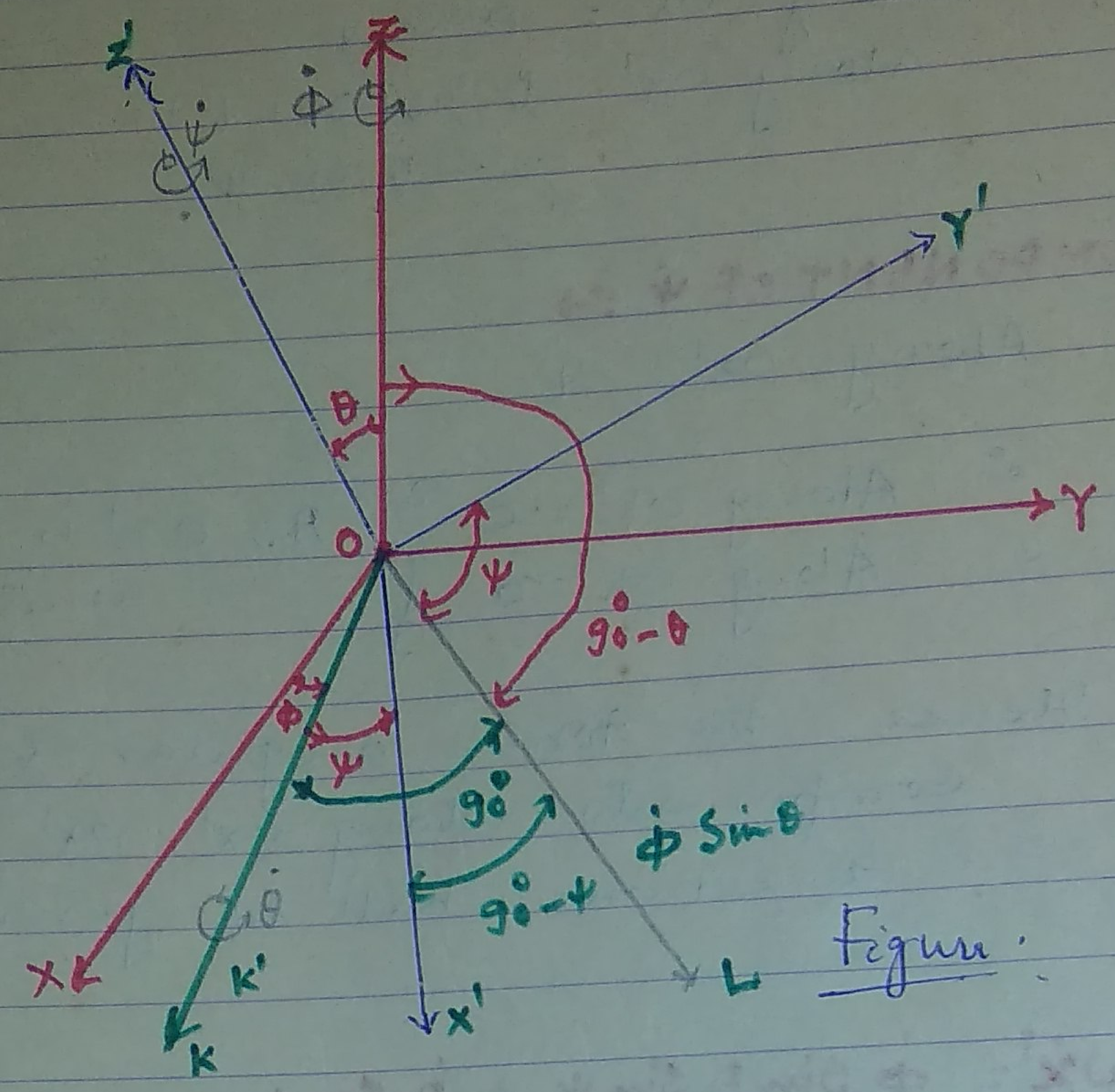

Figure.

Now let us draw a line OL in X'Y' plane at right angles to the line OK, The intersection of X–Y plane and X'Y' plane.

From the above figure, it is clear that

$\angle KOL = 90°$,

$\therefore \angle X'OL = 90° - \psi$

$\therefore \angle Y'OL = \psi$

Further, since $\angle ZOL = 90° - \theta$.

THE COMPONENT OF $\dot{\phi}$:→

Along $OZ' = \dot{\phi}\cos\theta$,

Along $OL = \dot{\phi}\sin\theta$,

$\therefore$ Along $OX' = \dot{\phi}\sin\theta\sin\psi$

& Along $OY' = \dot{\phi}\sin\theta\cos\psi$.

THE COMPONENT OF $\dot{\theta}$:→

Along $OZ' = 0$ ... Since OZ' is ⊥r to $X'Y'$ plane & hence to OK line,

Along $OX' = \dot{\theta} \cos\psi$

Along $OY' = \dot{\theta} \cos(90+\psi)$

$= -\dot{\theta} \sin\psi$.

THE COMPONENT OF $\dot{\psi}$:→

Along $OZ' = \dot{\psi}$

∴ Along $OX' = 0$
& Along $OY' = 0$ } As OZ' is ⊥r to OX' and OY'

Hence the total angular velocity components along OX', OY' & OZ' directions will be given by,

$$\boxed{\begin{aligned} \omega_{x'} &= \dot{\phi} \sin\theta \sin\psi + \dot{\theta} \cos\psi. \\ \omega_{y'} &= \dot{\phi} \sin\theta \cos\psi - \dot{\theta} \sin\psi \\ \omega_{z'} &= \dot{\phi} \cos\theta + \dot{\psi} \end{aligned}}$$

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