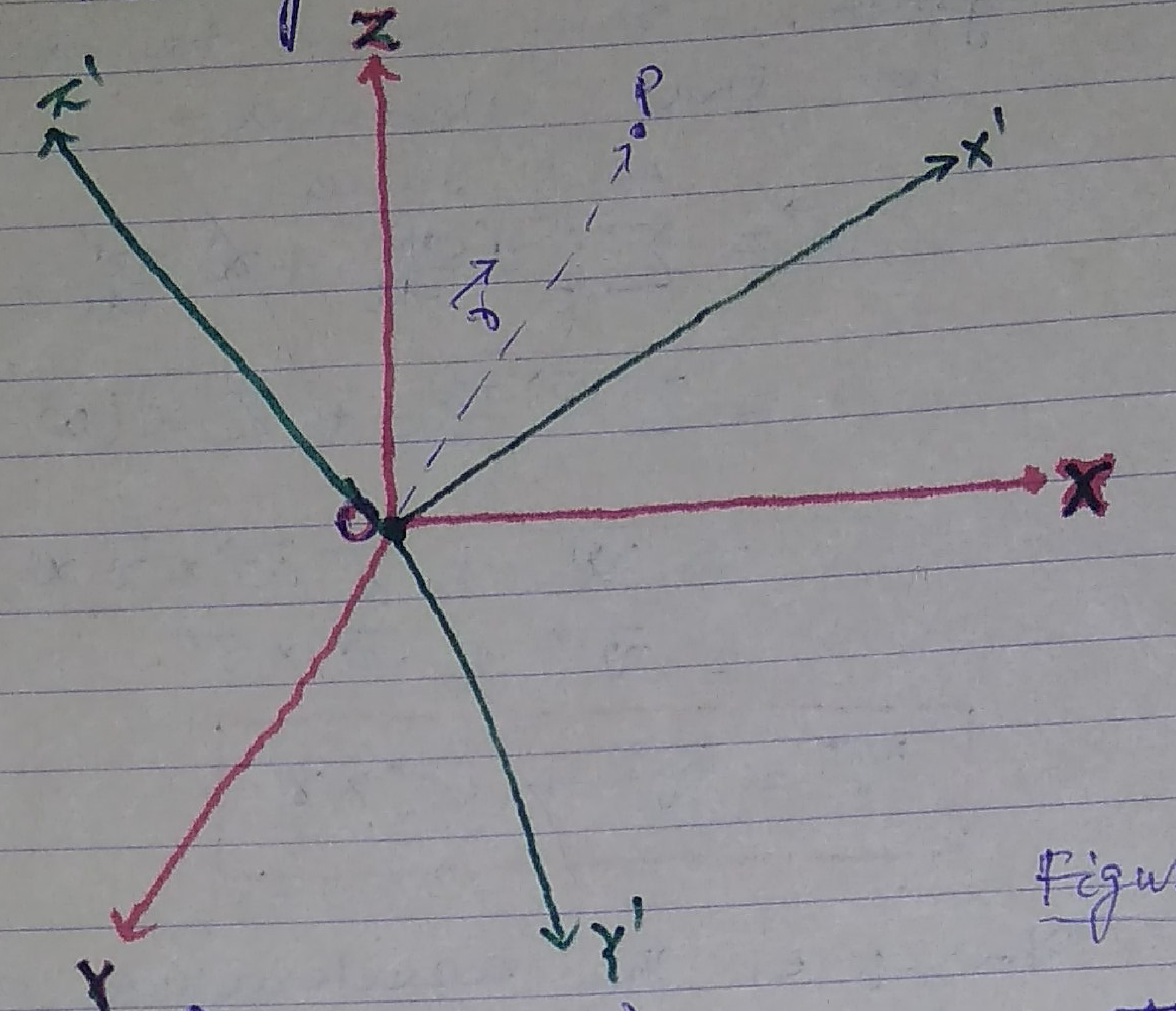
CORIOLIS FORCE –

ROTATING FRAME OF REFERENCE.

Let us consider a frame of reference x', y', z' rotating with a constant angular velocity "ω" with respect to a stationary frame of reference x, y, z, both frame having the same origin at O.



Figure

If $\vec{i}, \vec{j}$ & $\vec{k}$ represents the unit vectors along x, y & z directions and $\vec{i}', \vec{j}', \vec{k}'$ the unit vectors along x', y' & z' directions respectively. Then the position vector $\vec{r}$ of a particle P will be given by,

(i) For x', y', z' system,

$$\vec{r} = \vec{i}'x' + \vec{j}'y' + \vec{k}'z' \quad \text{and}$$

(ii) For x, y, z system,

$$\vec{r} = \vec{i}x + \vec{j}y + \vec{k}z.$$

Therefore the velocity of the particle at P will be given by,

(I) For an observer in x', y', z' system,

$$\vec{v}' = \frac{d\vec{r}}{dt} = \sum \frac{d}{dt}(\vec{i}'x') = \sum \vec{i}' \frac{dx'}{dt}$$

And (ii) For an observer in X, Y, Z system,

$$\vec{v} = \frac{d\vec{r}}{dt} = \sum \frac{d}{dt}(\vec{i'}x')$$

Since the unit vectors $\vec{i'}$, $\vec{j'}$ & $\vec{k'}$ are rotating for the observer in X, Y, Z System, they must be differentiable for that observer.

And Hence,

$$\vec{v} = \sum\left[\vec{i'}\frac{dx'}{dt} + x'\frac{d\vec{i'}}{dt}\right] \qquad \because v = \frac{d\vec{r}}{dt},\ \vec{v} = \vec{\omega}\times\vec{r}$$

$$= \sum \vec{i'}\frac{dx'}{dt} + \sum x'(\vec{\omega}\times\vec{i'})$$

$$= \vec{v'} + \sum \vec{\omega}\times\vec{i'}x'$$

$$= \vec{v'} + \vec{\omega}\times\vec{r}$$

$$\boxed{\vec{v} = \vec{v'} + \vec{\omega}\times\vec{r}} \quad \text{----- (a)}$$

Therefore, the acceleration of the Particle for the observer in X, Y, Z System will be given by,

$$\frac{d\vec{v}}{dt} = \frac{d\vec{v'}}{dt} + \frac{d}{dt}(\vec{\omega}\times\vec{r})$$

$$= \frac{d}{dt}\sum \vec{i'}\vec{v_x'} + \vec{\omega}\times\frac{d\vec{r}}{dt}$$

$$= \sum \vec{i'}\frac{d}{dt}v_x' + \sum v_x'\frac{d\vec{i'}}{dt} + \vec{\omega}\times\frac{d\vec{r}}{dt}$$

$$= \frac{d\vec{v'}}{dt} + \sum v_x'(\vec{\omega}\times\vec{i'}) + \vec{\omega}\times\frac{d\vec{r}}{dt}$$

$$= \frac{d\vec{v'}}{dt} + \vec{\omega}\times\sum \vec{i'}v_x' + \left(\vec{\omega}\times\frac{d\vec{r}}{dt}\right)$$

$$= \frac{d\vec{v'}}{dt} + (\vec{\omega}\times\vec{v'}) + \vec{\omega}\times\left[\vec{v'} + (\vec{\omega}\times\vec{r})\right]$$

Using eqn (a)

$$\therefore \boxed{\frac{d\vec{v}}{dt} = \frac{d\vec{v'}}{dt} + 2(\vec{\omega}\times\vec{v'}) + \vec{\omega}\times(\vec{\omega}\times\vec{r})} \quad \text{(b)}$$

If m is the mass of the Particle, then the force acting on it for the observer in X, Y, Z System

$$\vec{F} = m\frac{d\vec{v}}{dt}$$

$$= m\frac{d\vec{v'}}{dt} + 2m(\vec{\omega}\times\vec{v'}) + m\vec{\omega}\times(\vec{\omega}\times\vec{r})$$

$$= \vec{F'} + 2m(\vec{\omega}\times\vec{v'}) + m\vec{\omega}\times(\vec{\omega}\times\vec{r})$$

$$\therefore \boxed{\vec{F'} = \vec{F} - 2m(\vec{\omega}\times\vec{v'}) - m\vec{\omega}\times(\vec{\omega}\times\vec{r})} \quad \text{(c)}$$

In this expression the term $[-2m(\vec{\omega}\times\vec{v'})]$ is called Coriolis force and the term $[-m\vec{\omega}\times(\vec{\omega}\times\vec{r})]$ is called Centrifugal force. These two forces are fictitious forces acting on the particle which arise due to the rotation of the frame of reference. If there is no rotation, i.e. if $\vec{\omega} = 0$, then the two forces will disappear.

CORIOLIS FORCE :→ The coriolis force is therefore a fictitious force acting on a Particle when the Particle moves relative to a rotating frame of reference and which is mathematically defined by,

$$\vec{F_c} = -2m(\vec{\omega}\times\vec{v'})$$

Where $\vec{\omega}$ is the angular velocity of the rotatory reference frame S' and $\vec{v}$ is the velocity of the Particle relative to the system S'.

The $-ve$ sign in the expression $\vec{F_c}$ indicats that the corioli's force is in the direction opposit to that given by right hand screw rule.

Qu^n 1 →