DEVIATION OF FREELY FALLING BODY FROM THE VERTICAL

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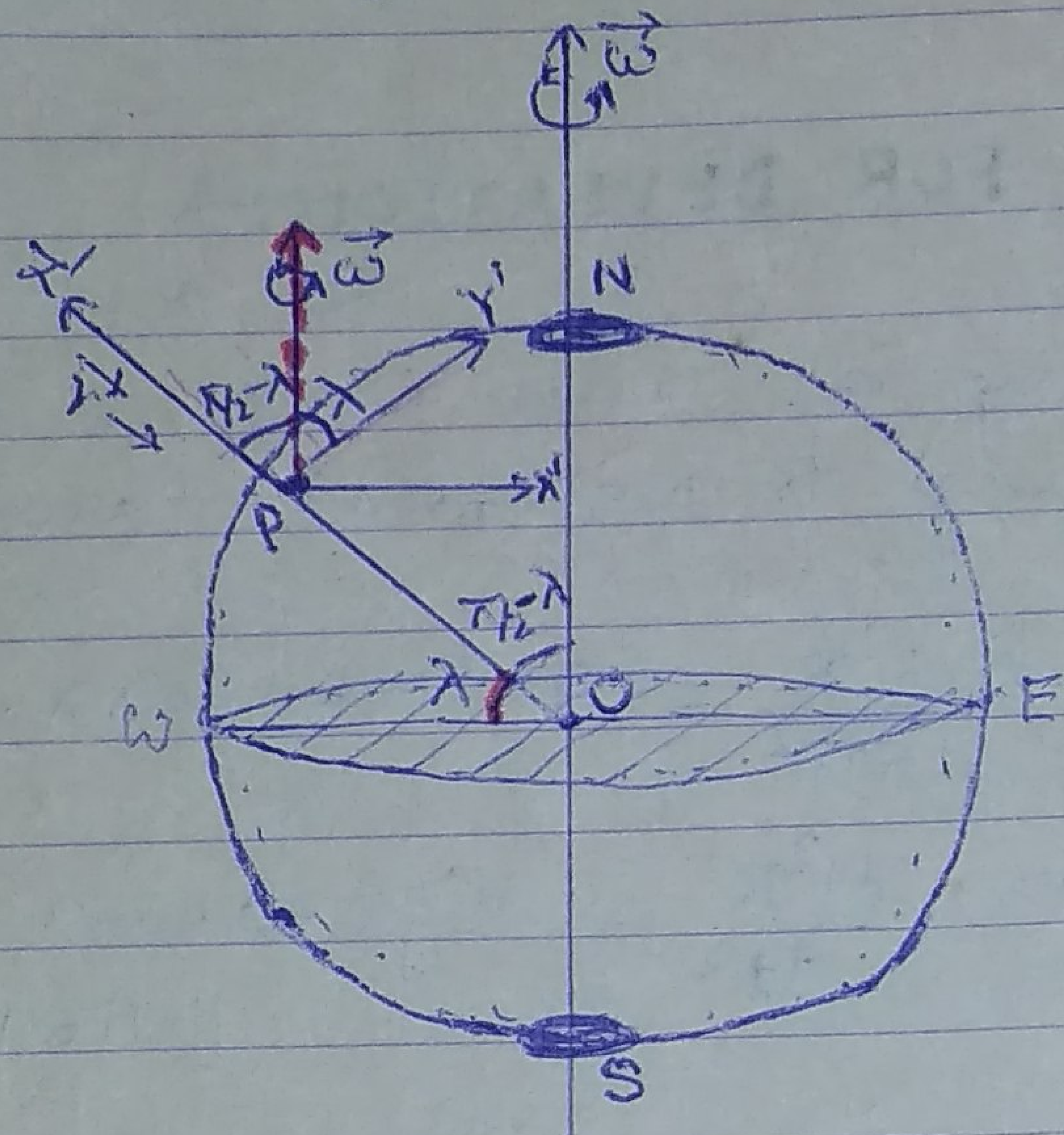
Let us Consider three perpendicular axes x', Y', Z' along east-north and vertical directions respectively, with the origin at P on the surface of the earth, rotating with an angular velocity $\vec{\omega}$ about its axis passing through the north and South pole.

Now let us suppose that a body of mass "m" is falling freely towards the point P and v is the velocity of the body at any instant "t".



Figure

If $\vec{i'}, \vec{j'}, \& \vec{k'}$ are the unit vectors along $x', Y' \& Z'$ directions respectively.

Then we will have;

$$\vec{\omega} = (\omega \cos\lambda)\vec{j'} + (\omega \sin\lambda)\vec{k'}$$

and $\vec{v} = -v\vec{k'}$;

$$\therefore \vec{\omega} \times \vec{v} = [(\omega \cos\lambda)\vec{j'} + (\omega \sin\lambda)\vec{k'}] \times (-v\vec{k'})$$

$$= -\omega v \cos\lambda \, \vec{i'}$$

Therefore, according to definition the coriolis force acting on the body at any instant t will be given by,

$$\vec{F_c} = -2m(\vec{\omega} \times \vec{v})$$

$$= (2m\omega v \cos\lambda)\,\hat{z} \quad \text{--- (1)}$$

This equation shows that the falling body experiences a coriolis force along east direction, due to which it suffers a deviation along the east.

EXPRESSION FOR DEVIATION:→

If x is the deviation distance of the body at any instant "t" Then according to Newton's 2nd law we can write,

$$\cancel{m}\frac{d^2x}{dt^2} = 2\cancel{m}v\omega\cos\lambda.$$

$$\text{or, } \frac{d^2x}{dt^2} = 2gt\,\omega\cos\lambda \quad \text{--- (2)}$$

[As $v = u + gt$
$= 0 + gt = gt$]

Integrating with respect to time, we get,

$$\frac{dx}{dt} = \cancel{2}g\omega\cos\lambda\,\frac{t^2}{\cancel{2}} + A$$

[Where A being an integration constant].

Since, at $t = 0$, $\frac{dx}{dt} = 0$, $\therefore A = 0$,

$$\therefore \frac{dx}{dt} = (g\omega\cos\lambda)t^2 \quad \text{--- (3)}$$

Further

Further integrating we get,

$x = g\omega \cos\lambda \frac{t^3}{3} + B$

Where B Being another integral constant.

Since at, $t=0$, $x=0$,

we get $B=0$,

And hence $x = (g\omega\cos\lambda)\frac{t^3}{3}$ --- (4)

If h is the vertical distance travelled by the body in time "t"

Then we will have,

$h = \frac{1}{2}gt^2$

or, $t = \sqrt{\frac{2h}{g}}$

Putting this value in eqn. (4)

$x = \frac{1}{3} g\omega \cos\lambda \left(\frac{2h}{g}\right)^{3/2}$ -- (5)

This is the expression for deviation of falling body from the vertical due to the rotation of the earth.

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THE END