Rigid rotator model (rigid diatomic molecule)

The diatomic molecule is supposed to be a system consisting of two atoms of masses m_1 and m_2 connected by a rigid rod of length r (bond length) and r_1 and r_2 be the distances of two atoms from the centre of gravity of the system.

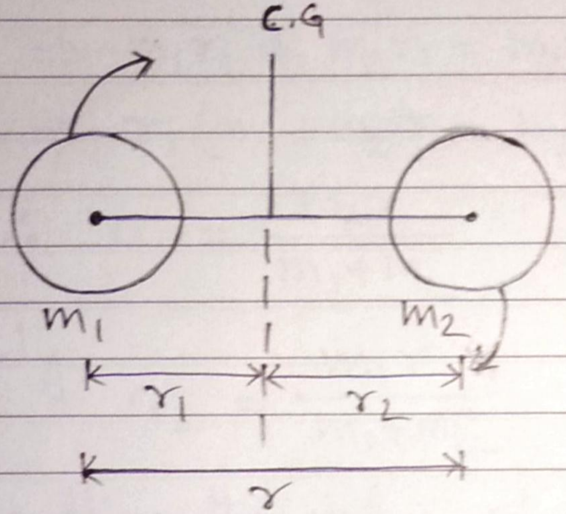

Fig: Rigid rotator model for a diatomic molecule.

Moment of inertia (I) - The molecule has a natural rotation about the axis through its centre of gravity. The moment of inertia of this system is given by.

$$I = m_1 r_1^2 + m_2 r_2^2 \qquad \text{--- (1)}$$

As the system is balanced about its centre of gravity, we can write

$$m_1 r_1 = m_2 r_2 \quad \text{--- (2)}$$

Also $$r = r_1 + r_2 \quad \text{--- (3)}$$

From eqn (2) and (3)

$$m_1 r_1 = m_2 (r - r_1)$$

$$\Rightarrow m_1 r_1 = m_2 r - m_2 r_1$$

$$\Rightarrow m_1 r_1 + m_2 r_1 = m_2 r$$

$$\Rightarrow r_1 (m_1 + m_2) = m_2 r$$

$$\therefore r_1 = \frac{m_2 r}{m_1 + m_2} \quad \text{--- (4)}$$

Similarly, $$r_2 = \frac{m_1 r}{m_1 + m_2} \quad \text{--- (5)}$$

On putting the value of r_1 and r_2 in eqn (1) we get,

$$I = m_1 \left(\frac{m_2 r}{m_1 + m_2}\right)^2 + m_2 \left(\frac{m_1 r}{m_1 + m_2}\right)^2$$

$$= \frac{m_1 m_2^2 r^2}{(m_1 + m_2)^2} + \frac{m_2 m_1^2 r^2}{(m_1 + m_2)^2}$$

$$= \frac{m_1 m_2^2 r^2 + m_2 m_1^2 r^2}{(m_1 + m_2)^2}$$

$$= \frac{m_1 m_2 r^2 \cancel{(m_2 + m_1)}}{(m_1 + m_2)^{\cancel{2}}}$$

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$$\therefore I = \frac{m_1 m_2 r^2}{m_1 + m_2}$$

$$\text{or } I = \frac{m_1 m_2}{m_1 + m_2} \cdot r^2$$

Now, $\frac{m_1 m_2}{m_1 + m_2} = \mu$ Known as reduced mass of the system.

$$\therefore \boxed{I = \mu r^2} \text{ --- } (6)$$

Eq$\underline{n}$ (6) defines moment of inertia in terms of atomic masses and bond length.