FOURIER TRANSFORM (17)

Problem (13) Find the finite cosine transform of $f(x)$ if (i) $f(x) = \frac{\pi}{3} - x + \frac{x^2}{2\pi}$

(ii) $f(x) = \sin nx$

Answer Since $F_c\{f(x)\} = \bar{f}_c(s) = \int_0^\pi f(x) \cos\frac{\pi s x}{\pi}\, dx$

$$= \int_0^\pi f(x) \cos(sx)\, dx$$

(i) $F_c\{f(x)\} = F_c\left\{\frac{\pi}{3} - x + \frac{x^2}{2\pi}\right\}$

$$= \int_0^\pi \left(\frac{\pi}{3} - x + \frac{x^2}{2\pi}\right) \cos sx\, dx$$

$$= \left\{\left(\frac{\pi}{3} - x + \frac{x^2}{2\pi}\right) \cdot \frac{\sin sx}{s}\right\}_{x=0}^{\pi} - \int_0^\pi \left(-1 + \frac{x}{\pi}\right) \frac{\sin sx}{s}\, dx$$

$$= (0 - 0) - \frac{1}{s}\left[\left(-1 + \frac{x}{\pi}\right)\frac{-\cos sx}{s} - \int \frac{1}{\pi} \frac{-\cos sx}{s}\, dx\right]_{x=0}^{\pi}$$

$$= +\frac{1}{s}\left[\left(-1 + \frac{x}{\pi}\right)\frac{\cos sx}{s}\right]_0^\pi - \frac{1}{\pi s^2}\left[\frac{\sin sx}{s}\right]_{x=0}^{\pi}$$

$$= \frac{1}{s^2}\left[0 + 1\right] - 0 = \frac{1}{s^2} \text{ if } s = 1, 2, 3, \ldots$$

Ans

(ii) $F_c\{f(x)\} = F_c\{\sin nx\} = \int_0^\pi \sin nx \cos\frac{\pi s x}{\pi}\, dx$

$$= \frac{1}{2}\int_0^\pi 2 \sin nx \cos sx\, dx$$

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$$= \frac{1}{2}\int_0^\pi \left[\sin(n+s)x + \sin(n-s)x\right] dx$$

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$$F_c\{\sin nx\} = -\frac{1}{2}\left[\frac{\cos(n+s)x}{n+s} + \frac{\cos(n-s)x}{n-s}\right]_{x=0}^{\pi}$$

$$= -\frac{1}{2}\left[\frac{\cos(n+s)\pi}{n+s} - \frac{1}{n+s} + \frac{\cos(n-s)\pi}{n-s} - \frac{1}{n-s}\right]$$

$$= -\frac{1}{2}\left[\frac{(-1)^{n+s}}{n+s} + \frac{(-1)^{n-s}}{n-s} - \frac{1}{n+s} - \frac{1}{n-s}\right]$$

$$= \begin{cases} 0 & \text{, if } n+s \text{ is even} \\ \frac{1}{2}\left[\frac{2}{n+s} + \frac{2}{n-s}\right] & \text{, if } n+s \text{ is odd} \end{cases}$$

$$= \begin{cases} 0 & \text{, if } n+s \text{ is even} \\ \frac{2n}{n^2-s^2} & \text{, if } n+s \text{ is odd.} \end{cases}$$

Ans.

Problem (14) Find the finite Fourier Cosine transform of $f(x)$, where

$$f(x) = \begin{cases} 1 & \text{, for } 0 < x < \frac{\pi}{2} \\ -1 & \text{, for } \frac{\pi}{2} < x < \pi \end{cases}$$

Answer $\bar{f}_c(p) = \int_0^{\pi} f(x) \cos px \, dx$

$$= \int_0^{\pi/2} f(x) \cos px \, dx + \int_{\pi/2}^{\pi} f(x) \cos px \, dx$$

$$= \int_0^{\pi/2} 1 \cdot \cos px \, dx + \int_{\pi/2}^{\pi} (-1) \cos px \, dx$$

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$$\bar{f}_c(p) = \left[\frac{\sin px}{p}\right]_0^{\pi/2} - \left[\frac{\sin px}{p}\right]_{\pi/2}^{\pi}$$

$$= \left(\frac{\sin p\frac{\pi}{2}}{p} - 0\right) - \left(\frac{\sin p\pi}{p} - \frac{\sin p\frac{\pi}{2}}{p}\right)$$

$$= \frac{2\sin p\frac{\pi}{2}}{p} - \frac{\sin p\pi}{p}$$

$$= \frac{2}{p}\sin\left(\frac{p\pi}{2}\right), \text{ where } p = 1, 2, 3, \ldots$$

When $p = 0$, then

$$\bar{f}_c(p) = \int_0^{\pi/2} 1 \cdot dx + \int_{\pi/2}^{\pi} (-1)\, dx$$

$$= [x]_0^{\pi/2} - [x]_{\pi/2}^{\pi}$$

$$= \left(\frac{\pi}{2} - 0\right) - \left(\pi - \frac{\pi}{2}\right) = \frac{\pi}{2} - \frac{\pi}{2} = 0$$

$$\therefore \bar{f}_c(p) = \begin{cases} \frac{2}{p}\sin\frac{p\pi}{2}, & p = 1, 2, 3, \ldots \\ 0, & p = 0 \end{cases}$$

Ans

Problem (15) Find finite cosine transform of $f(x) = x,\ 0 < x < \pi$.

Answer $f_c(x) = \int_0^{\pi} f(x)\cos\frac{s\pi x}{\pi}\, dx = \int_0^{\pi} x\cos(sx)\, dx$

$$= \left[x\frac{\sin(sx)}{s}\right]_0^{\pi} - \int_0^{\pi} 1 \cdot \frac{\sin(sx)}{s}\, dx$$

$$= 0 - \frac{1}{s}\left[-\frac{\cos sx}{s}\right]_0^{\pi} = \frac{1}{s^2}\left[\cos s\pi - \cos 0\right]$$

$$= \frac{1}{s^2}\left[(-1)^s - 1\right]$$

Ans