



Scientific Computing

Topic:- Gauss Elimination Method

part – III

Karim City College
Department of Computer Application
Faculty: Prof. Yashmin Banu
BCA- Sem-3

EXAMPLE 2.2.11 *Solve the linear system by Gauss elimination method.*

$$y + z = 2$$

$$2x + 3z = 5$$

$$x + y + z = 3$$

$$\begin{bmatrix} 0 & 1 & 1 & 2 \\ 2 & 0 & 3 & 5 \\ 1 & 1 & 1 & 3 \end{bmatrix}.$$

Solution: In this case, the augmented matrix is along the following steps.

The method proceeds

1. Interchange 1st and 2nd equation (or R_{12}).

$$\begin{array}{rcl} 2x + 3z & = & 5 \\ y + z & = & 2 \\ x + y + z & = & 3 \end{array} \qquad \begin{bmatrix} 2 & 0 & 3 & 5 \\ 0 & 1 & 1 & 2 \\ 1 & 1 & 1 & 3 \end{bmatrix}.$$

2. Divide the 1st equation by 2 (or $R_1(1/2)$).

$$\begin{array}{rcl} x + \frac{3}{2}z & = & \frac{5}{2} \\ y + z & = & 2 \\ x + y + z & = & 3 \end{array} \qquad \begin{bmatrix} 1 & 0 & \frac{3}{2} & \frac{5}{2} \\ 0 & 1 & 1 & 2 \\ 1 & 1 & 1 & 3 \end{bmatrix}.$$

3. Add -1 times the 1st equation to the 3rd equation (or $R_{31}(-1)$).

$$\begin{array}{rcl} x + \frac{3}{2}z & = & \frac{5}{2} \\ y + z & = & 2 \\ y - \frac{1}{2}z & = & \frac{1}{2} \end{array} \qquad \begin{bmatrix} 1 & 0 & \frac{3}{2} & \frac{5}{2} \\ 0 & 1 & 1 & 2 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}.$$

4. Add -1 times the 2nd equation to the 3rd equation (or $R_{32}(-1)$).

$$\begin{array}{rcl} x + \frac{3}{2}z & = & \frac{5}{2} \\ y + z & = & 2 \\ -\frac{3}{2}z & = & -\frac{3}{2} \end{array} \qquad \begin{bmatrix} 1 & 0 & \frac{3}{2} & \frac{5}{2} \\ 0 & 1 & 1 & 2 \\ 0 & 0 & -\frac{3}{2} & -\frac{3}{2} \end{bmatrix}.$$

5. Multiply the 3rd equation by $\frac{-2}{3}$ (or $R_3(-\frac{2}{3})$).

$$\begin{array}{rcl} x + \frac{3}{2}z & = & \frac{5}{2} \\ y + z & = & 2 \\ z & = & 1 \end{array} \qquad \begin{bmatrix} 1 & 0 & \frac{3}{2} & \frac{5}{2} \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

The last equation gives $z = 1$, the second equation now gives $y = 1$. Finally the first equation gives $x = 1$. Hence the set of solutions is $(x, y, z)^t = (1, 1, 1)^t$, A UNIQUE SOLUTION.

EXAMPLE 2.2.13 *Solve the linear system by Gauss elimination method.*

$$x + y + z = 3$$

$$x + 2y + 2z = 5$$

$$3x + 4y + 4z = 12$$

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 1 & 2 & 2 & 5 \\ 3 & 4 & 4 & 12 \end{bmatrix}$$

Solution: In this case, the augmented matrix is and the method proceeds as follows:

1. Add $^{-1}$ times the first equation to the second equation.

$$\begin{array}{rcl} x + y + z & = & 3 \\ y + z & = & 2 \\ 3x + 4y + 4z & = & 12 \end{array} \quad \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 1 & 2 \\ 3 & 4 & 4 & 12 \end{bmatrix}.$$

2. Add $^{-3}$ times the first equation to the third equation.

$$\begin{array}{rcl} x + y + z & = & 3 \\ y + z & = & 2 \\ y + z & = & 3 \end{array} \quad \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 3 \end{bmatrix}.$$

3. Add $^{-1}$ times the second equation to the third equation

$$\begin{array}{rcl} x + y + z & = & 3 \\ y + z & = & 2 \\ 0 & = & 1 \end{array} \quad \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The third equation in the last step is

$$0x + 0y + 0z = 1.$$

This can never hold for any value of x, y, z . Hence, the system has NO SOLUTION.