## Standard results.

$L\{1\} = \frac{1}{s} \quad s > 0$

$L\{t^n\} = \frac{\Gamma(n+1)}{s^{n+1}}$ if $s > 0$ & $n > -1$ ($\Gamma_{n+1} = \lfloor n$ if $n$ is +ve integer.)

$L\{e^{at}\} = \frac{1}{s-a}$ if $s > a$

$L\{\cos at\} = \frac{s}{s^2+a^2}$

$L\{\sin at\} = \frac{a}{s^2+a^2}$

$L\{e^{at}.t^n\} = \frac{\lfloor n}{(s-a)^{n+1}}$ if $n$ is positive integer

$L\{\cosh at\} = \frac{s}{s^2-a^2}$

$L\{\sinh at\} = \frac{a}{s^2-a^2}$

$L\{t^n F(t)\} = (-1)^n \frac{d^n f(s)}{ds^n}$ for $n = 1, 2, 3, \ldots$

$L\left\{\frac{F(t)}{t}\right\} = \int_s^\infty f(s)\,ds.$

$L\{F^{(n)}(t)\} = s^n f(s) - s^{n-1} F(0) - s^{n-2} F'(0) - \cdots$
$\cdots - sF^{n-2}(0) - F^{n-1}(0)$

$L\left\{\int_0^t F(u)\,du\right\} = \frac{1}{s} f(s)$

Note: $f(s) = L\{F(t)\}$

When $t=1$ $\quad u=0$

When $t=\infty$ $\quad u=\infty$

$$= 0 + \int_0^\infty e^{-s(u+1)} \cdot u^2\,du$$

$$= e^{-s}\int_0^\infty e^{-su} \cdot u^2\,du$$

$$= e^{-s}\int_0^\infty e^{-st} t^2\,dt$$

$$= e^{-s} L\{t^2\}$$

$$= \frac{e^{-s} \cdot \lfloor 2}{s^3} \quad \text{Ans}$$

Q: $L\left\{\frac{\sin at}{t}\right\}$

Sol$^n$ We have $L\left\{\frac{\sin at}{t}\right\} = \int_s^\infty L\{\sin at\}\,ds$

$$= \int_s^\infty \frac{a}{s^2+a^2}\,ds$$

$$= \left[\tan^{-1}\frac{s}{a}\right]_s^\infty$$

$$= \frac{\pi}{2} - \tan^{-1}\frac{s}{a}$$

$$= \cot^{-1}\frac{s}{a}. \quad \text{Ans}$$

Note: $L\left\{\frac{\cos at}{t}\right\}$ does not exist

Q: $L\{(t^5 e^{3t})\}$

Sol$^n$ We have $L\{t^5 e^{3t}\} = \dfrac{5!}{(s-3)^{5+1}}$

$$= \frac{5!}{(s-3)^6}$$

Q: $L\{3e^{3t} + 5t^4 - 4\cos 3t\}$

Sol$^n$ $L\{3e^{3t} + 5t^4 - 4\cos 3t\}$

$$= 3L\{e^{3t}\} + 5L\{t^4\} - 4L\{\cos 3t\}$$

$$= 3\cdot\frac{1}{s-3} + 5\cdot\frac{4!}{s^5} - 4\frac{s}{s^2+3^2}$$

$$= \frac{3}{s-3} + \frac{120}{s^5} - \frac{4s}{s^2+9} \quad \text{Ans}$$

Q: Find Laplace transform of $F(t)$ given by

$$F(t) = \begin{cases} (t-1)^2 & \text{if } t>1 \\ 0 & \text{if } 0<t\leq 1 \end{cases}$$

Sol$^n$ $L\{F(t)\} = \int_0^\infty e^{-st} F(t)\, dt$

$$= \int_0^1 e^{-st} F(t)\, dt + \int_1^\infty e^{-st}(t-1)^2 dt.$$

$$= \int_0^1 e^{-st} \times 0 \cdot dt + \int_1^\infty e^{-st}\cdot(t-1)^2 dt$$

Let in second integral.

$t-1 = u$

$dt = du$

Q: Prove that $\int_0^{\infty} e^{-t} \cdot \frac{\sin t}{t} \, dt \; \frac{\pi}{4}$.

Sol$^n$ We know that

$$L\left\{\frac{\sin at}{t}\right\} = \cot^{-1}\left(\frac{s}{a}\right)$$

show

$$\Rightarrow \int_0^{\infty} e^{-st} \cdot \frac{\sin at}{t} \, dt = \cot^{-1}\frac{s}{a}$$

Putting $s=1$ & $a=1$

$$\Rightarrow \int_0^{\infty} e^{-t} \cdot \frac{\sin t}{t} = \cot^{-1}(1)$$

$$\Rightarrow \int_0^{\infty} e^{-t} \frac{\sin t}{t} = \frac{\pi}{4}$$

Q: Prove that $\int_0^{\infty} \frac{\sin t}{t} \, dt = \frac{\pi}{2}$.

Sol$^n$ We know that

$$L\left\{\frac{\sin at}{t}\right\} = \cot^{-1}\left(\frac{s}{a}\right)$$

$$\Rightarrow \int_0^{\infty} e^{-st} \cdot \frac{\sin at}{t} \, dt = \cot^{-1}\left(\frac{s}{a}\right)$$

Putting $a=1$ & $s=0$

$$\Rightarrow \int_0^{\infty} e^{0} \cdot \frac{\sin t}{t} \, dt = \cot^{-1}\left(\frac{0}{a}\right)$$

$$\Rightarrow \int_0^{\infty} \frac{\sin t}{t} \, dt = \cot^{-1} 0$$

$$\Rightarrow \int_0^{\infty} \frac{\sin t}{t} \, dt = \frac{\pi}{2}$$