

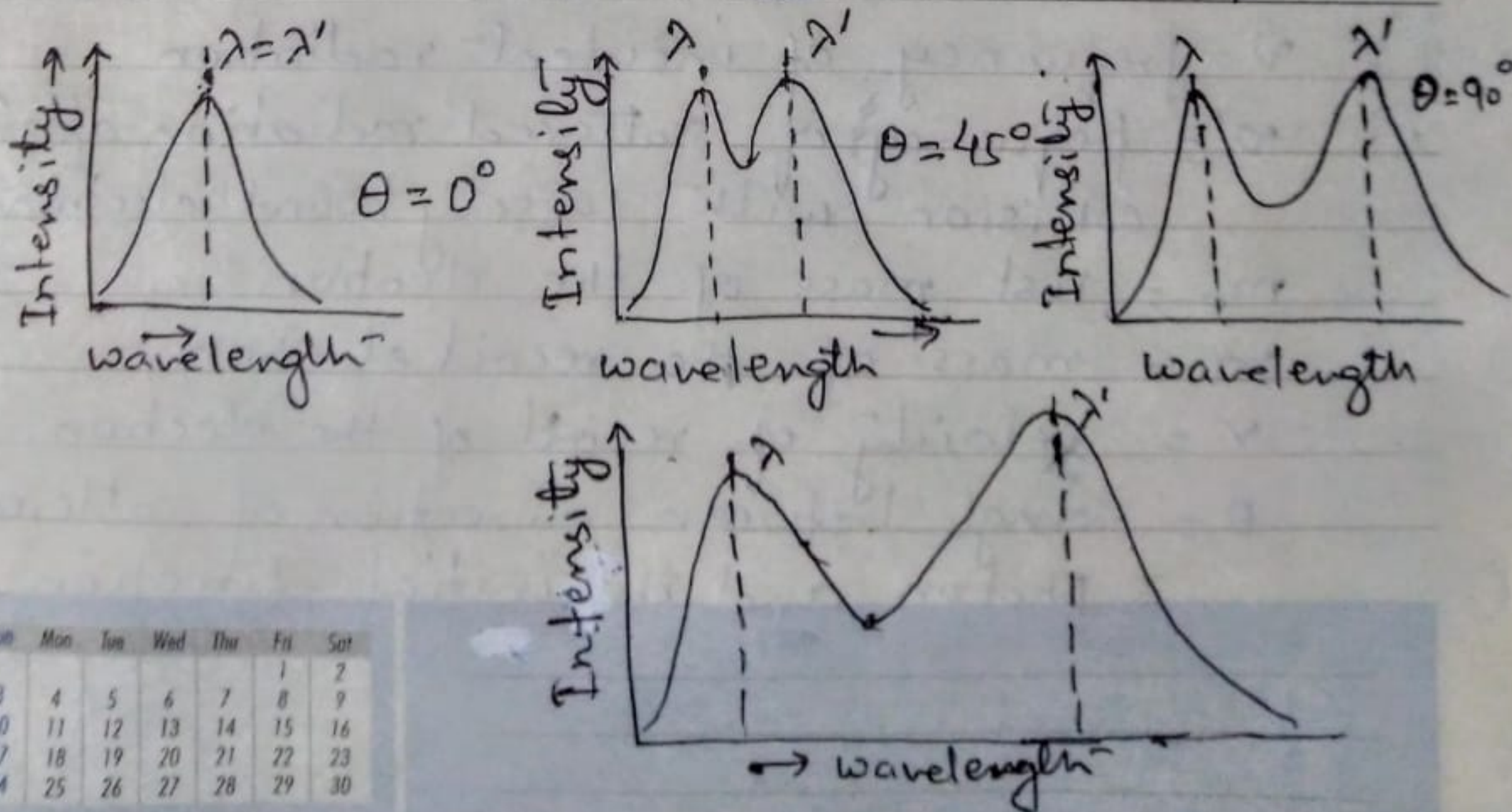
24thCompton effect-

When a beam of X-rays of sharply defined frequency is scattered by a light element like carbon, the scattered radiation consists of two components, one of larger wavelength than the incident radiation and the second of the same wavelength as the incident radiation.

25th

This change in frequency or wavelength of X-ray due to scattering (i.e., incoherent scattering) is called 'Compton Effect'.

The experimentally obtained graph of intensity versus wavelength of scattered X-ray for different angles of scattering θ is shown in fig 1. The wavelength of the scattered radiation depends on

26th

April 2005

Sun	Mon	Tue	Wed	Thu	Fri	Sat
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

28 mon

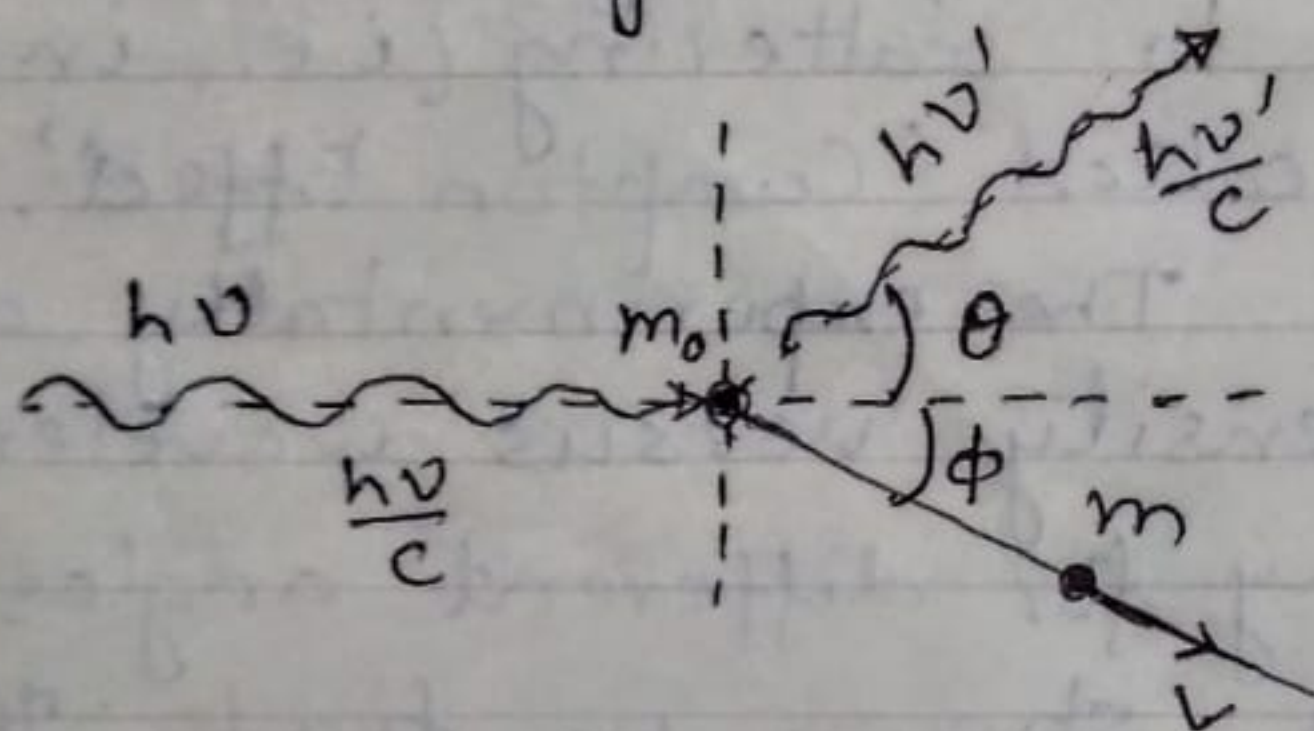
the angle of scattering and the wavelength of the incident wave. The spectrum consists of two peaks.

The scattering can be explained by taking photons as particles colliding elastically with loosely bound electrons.

Expression for Compton shift

29 tue

Let a photon of radiation be incident on a loosely bound electron at rest. Let



30 wed

ν = frequency of incident radiation

ν' = frequency of scattered radiation after collision with loosely bound electron.

m_0 = rest mass of the electron

m = mass of the recoil electron

v = velocity of recoil of the electron

θ = angle between direction of scattered photon and the initial direction

31 thu

ϕ = angle between direction of recoil electron and the initial direction.

Conservation of energy gives:

$$h\nu + m_0 c^2 = h\nu' + mc^2 \quad \text{--- (1)}$$

where $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ --- (2)

Conservation of momentum along and perpendicular to the direction of incident radiation gives:

1 fri

$$\frac{h\nu}{c} = \frac{h\nu'}{c} \cos\theta + mv \cos\phi \quad \text{--- (3)}$$

$$0 = \frac{h\nu'}{c} \sin\theta - mv \sin\phi \quad \text{--- (4)}$$

From equations (3) and (4) we get:-

$$\begin{aligned} (mv \cos\phi)^2 + (mv \sin\phi)^2 &= \left(\frac{h\nu}{c} - \frac{h\nu'}{c} \cos\theta \right)^2 + \left(\frac{h\nu'}{c} \sin\theta \right)^2 \\ &= \frac{h^2}{c^2} (\nu^2 - 2\nu\nu' \cos\theta + \nu'^2 \cos^2\theta + \nu'^2 \sin^2\theta) \end{aligned}$$

2 sat

$$\Rightarrow m^2 v^2 (\cos^2\phi + \sin^2\phi) = \frac{h^2}{c^2} (\nu^2 - 2\nu\nu' \cos\theta + \nu'^2)$$

$$\Rightarrow m^2 v^2 c^2 = h^2 (\nu^2 - 2\nu\nu' \cos\theta + \nu'^2) \quad \text{--- (5)}$$

From equation (1) we have

$$mc^2 = h(\nu - \nu') + m_0 c^2$$

Squaring it- we get:-

$$m^2 c^4 = h^2 (\nu^2 - 2\nu\nu' + \nu'^2) + 2h(\nu - \nu') m_0 c^2 + m_0^2 c^4 \quad \text{--- (6)}$$

4 mon

subtracting equation (5) from equation (6)

we get:

$$m^2 c^2 (c^2 - v^2) = h^2 (v^2 - 2vv' + v'^2 - v^2 + 2vv' \cos \theta - v'^2) + 2h(v - v') m_0 c^2 + m_0^2 c^4$$

$$\Rightarrow \frac{m_0^2}{1 - \frac{v^2}{c^2}} c^2 (c^2 - v^2) = h^2 (2vv' \cos \theta - 2vv') + 2h(v - v') m_0 c^2 + m_0^2 c^4$$

(using eqⁿ. 2)

5 tue $\Rightarrow m_0^2 c^4 = 2h^2 vv' (\cos \theta - 1) + 2h(v - v') m_0 c^2 + m_0 c^4$

$$\Rightarrow 2h(v - v') m_0 c^2 = 2h^2 vv' (1 - \cos \theta)$$

$$\Rightarrow \frac{v - v'}{vv'} = \frac{h}{m_0 c^2} (1 - \cos \theta)$$

$$\Rightarrow \frac{1}{v'} - \frac{1}{v} = \frac{h}{m_0 c^2} (1 - \cos \theta)$$

$$\Rightarrow \frac{c}{v'} - \frac{c}{v} = \frac{h}{m_0 c} (1 - \cos \theta)$$

6 wed $\Rightarrow \lambda' - \lambda = \frac{h}{m_0 c} (1 - \cos \theta)$

where $\lambda = \frac{c}{v}$ = Wavelength of incident photon

$\lambda' = \frac{c}{v'}$ = wavelength of scattered photon

\therefore The change in wavelength = $\Delta \lambda = \frac{h}{m_0 c} (1 - \cos \theta)$

This is the required expression for Compton shift.

→ X →

Notes