Ex Using Heaviside's expansion formula find

$$L^{-1}\left\{\frac{2p^2+5p-4}{p^3+p^2-2p}\right\}$$

Sol^n Here $F(p) = 2p^2+5p-4$

$G(p) = p^3+p^2-2p$

Obviously $\deg F(p) < \deg G(p)$

Also $G(p) = p(p^2+p-2)$

$= p(p-1)(p+2)$

$\Rightarrow$ Zeros of $G(p)$ are $0, 1, -2$.

$G'(p) = 3p^2+2p-2$

At 0 $\quad F(0) = -4 \quad \& \quad G'(0) = -2$

At 1 $\quad F(1) = 3 \quad \& \quad G'(1) = 3$

At -2 $\quad F(-2) = -6 \quad \& \quad G'(-2) = 6$

We know that

$$L\left\{\frac{F(p)}{G(p)}\right\} = \sum_{i=1}^{3} \frac{F(\alpha_i)}{G'(\alpha_i)} e^{t\alpha_i}$$ when α_i is zero of $G(p)$, $i=1,2,3$.

$$= \frac{F(0)}{G'(0)} e^{t\cdot 0} + \frac{F(1)}{G'(1)} \cdot e^{t\cdot 1} + \frac{F(-2)}{G'(-2)} \cdot e^{t\cdot(-2)}$$

$$= \frac{-4}{-2} e^0 + \frac{3}{3} e^t + \frac{-6}{6} \cdot e^{-2t}$$

$$= 2 + e^t - e^{-2t}$$

Ans

(2)

We can solve previous problem by partial fraction method

Ex find $L^{-1}\left\{\dfrac{2p^2+5p-4}{p^3+p^2-2p}\right\}$

Solⁿ We have $\dfrac{2p^2+5p-4}{p^3+p^2-2p} = \dfrac{2p^2+5p-4}{p(p-1)(p+2)}$

Let $\dfrac{2p^2+5p-4}{p(p-1)(p+2)} = \dfrac{A}{p} + \dfrac{B}{p-1} + \dfrac{C}{p+2}$ —— (1)

$\Rightarrow \dfrac{2p^2+5p-4}{p(p-1)(p+2)} = \dfrac{A(p-1)(p+2)+Bp(p+2)+Cp(p-1)}{p(p-1)(p+2)}$

$\Rightarrow 2p^2+5p-4 = A(p-1)(p+2)+Bp(p+2)+Cp(p-1)$ —— (2)

Putting in (2) $p=0$ $\quad -4 = A(-2)$ $\quad A=2$

$p=1$ $\quad 3 = B3$ $\quad B=1$

$p=-2$ $\quad -6 = C(6)$ $\quad C=-1$

Putting in (1)

$\Rightarrow \dfrac{2p^2+5p-4}{p(p-1)(p+2)} = \dfrac{2}{p} + \dfrac{1}{p-1} - \dfrac{1}{p+2}$

$\Rightarrow L^{-1}\left\{\dfrac{2p^2+5p-4}{p(p-1)(p+2)}\right\} = 2L^{-1}\left\{\dfrac{1}{p}\right\} + L^{-1}\left\{\dfrac{1}{p-1}\right\} - L^{-1}\left\{\dfrac{1}{p+2}\right\}$

$= 2(1) + e^{t} - e^{-2t}$.

$= 2 + e^{t} - e^{-2t}$

Ans

Application of Laplace Transform:

Consider differential equation

$$a\frac{d^2x}{dt^2} + b\frac{dx}{dt} + x = F(t) \quad \text{--- (1)}$$

Case I If a and b are constants

We know that $L\{F^{(n)}(t)\} = s^n f(s) - s^{n-1}F(0) - s^{n-2}F'(0)$

$- \cdots - F^{n-1}(0)$.

Taking Laplace transform (1) eqn (1)

$$aL\{x''\} + bL\{x'\} + L\{x\} = L\{F(t)\} \text{ ---}$$

$$\Rightarrow a\{s^2\bar{x}(s) - s\,x(0) - x'(0)\} + b\{s\bar{x}(s) - x(0)\} + \bar{x}(s) = f(s) \text{ --- (2)}$$

where $L(x) = \bar{x}(s)$

Now solution (1) obtained by taking inverse Laplace transform of $\bar{x}(s)$ ($\bar{x}(s)$ is obtained from (2)).

Case II If a & b are functions of t. ie specially of the form $t^m\frac{d^2x}{dt^2} + t\frac{dx}{dt} + x = F(t)$

We will us following result.

$$L\left\{t^m \frac{d^n x}{dt^n}\right\} = (-1)^m \frac{d^m}{ds^m}\left\{L\left(\frac{d^n x}{dt^n}\right)\right\}$$

and finding $\bar{x}(s)$ and then finding the inverse laplace transform of $\bar{x}(s)$

ie soln is $L^{-1}\{\bar{x}(s)\}$