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Tensor calculus (20)

Some definitions

1. Covariant derivative of a scalar: Covariant derivative of a scalar ϕ w.r.t. x^i is defined as its ordinary partial derivative w.r.t. x^i and is denoted by $\phi_{,i}$

Thus $\phi_{,i} = \frac{\partial \phi}{\partial x^i}$

Evidently $\phi_{,i} = \frac{\partial \phi}{\partial x^i} = \nabla \phi$.

2. Divergence of a vector: The divergence of a contravariant vector A^i is defined as the contraction of its covariant derivative and is denoted by $\operatorname{div} A^i$. Thus

$$\operatorname{div} A^i = A^i_{,i}$$

The divergence of a covariant vector A_i is denoted by $\operatorname{div} A_i$ and is defined as

$$\operatorname{div} A_i = g^{ik} A_{J,K}$$

It can be shown that $\operatorname{div} A_i = \operatorname{div} A^i$

Proof: $\operatorname{div} A_i = g^{JK} A_{J,K} = (g^{JK} A_J)_{,k}$

$$= A^k_{,k} = \operatorname{div} A^k = \operatorname{div} A^i$$

$\therefore \operatorname{div} A_i = \operatorname{div} A^i$ Proved

3. Covariant Constants:

The covariant derivatives of the tensors g^{ij}, g_{ij}, g^i_j vanish identically.

It means these tensors behave as constants in covariant differentiation. Hence these tensors are defined as covariant constant.

4. Curl of a vector: Curl of a vector A_i, denoted by curl A_i or rot A_i is defined as $\text{rot}(A_i) = \text{curl}\, A_i = A_{i,j} - A_{j,i}$

i.e. $\text{curl}\, A_i = A_{i,j} - A_{j,i} = \frac{\partial A_i}{\partial x^j} - \frac{\partial A_j}{\partial x^i}$

5. Cross product of two vectors: The cross product of two vectors u and v, denoted by $u \times v$ and is defined as

$$u \times v = u_i v_j - u_j v_i$$

Evidently $u \times v \neq v \times u$.

6. Intrinsic Derivative: Consider a tensor $A^{ij\ldots k}_{ab\ldots c}$ defined along a curve $x^i = x^i(t)$ of parameter t. The intrinsic derivative is defined by the equation

$$\frac{\delta A^{ij\ldots k}_{ab\ldots c}}{\delta t} = A^{ij\ldots k}_{ab\ldots c,p} \frac{dx^p}{dt}.$$

It means that the intrinsic derivative of a tensor is a tensor of the same rank and similar character as the original tensor.