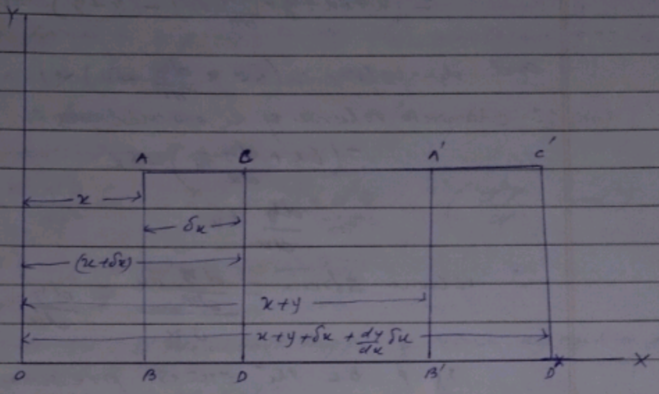
Velocity of Longitudinal wave in a gaseous medium:-



Let in a gaseous medium imagine a cylinder of unit cross-sectional area. Let the longitudinal wave propagated along OX from left to right.

Let AB & CD be the two plane at distance x and $(x+\delta x)$ from O, perpendicular to the direction of wave propagation. So that the length of the gaseous layer covered by the two surfaces will be δx and volume will be $\delta x \times 1 = \delta x$.

Let the planes AB and CD be displaced to the positions A'B' and C'D' respectively. If y be the displacement of AB, then distance of A'B' from O will be $(x+y)$. So the rate of change of displacement in the x direction will be $\frac{dy}{dx}$. So the change of displacement for a distance δx betn AB and CD $= \frac{dy}{dx}\delta x$. Hence the displacement of the plane $CD = y + \frac{dx}{dx}\delta x$ and the distance of the plane C'D' from $O = x+\delta x+y+\frac{dy}{dx}\delta x$

Now, the length of the gaseous layer covered by the planes A'B' and C'D'

$$= \left(x + \delta x + y + \frac{dy}{dx}\delta x\right) - (x + y) = \delta x + \frac{dy}{dx}\delta x$$

and its volume $= \left(\delta x \times \frac{dy}{dx} + \delta x\right) \times 1$

$\therefore$ change in volume of the enclosed gas

$$= \left(\delta x + \frac{dy}{dx}\delta x\right) - \delta x$$

$$= \frac{dy}{dx}\delta x$$

$\therefore$ volume strain $= \dfrac{\frac{dy}{dx}\delta x}{\delta x} = \dfrac{dy}{dx}$

If P be the excess pressure acting on the plane AB due to longitudinal wave, then coefficient of volume strain

$$K = -\frac{P}{dy/dx}$$ (− sign shows on increasing pressure volume decreases)

$$P = -K\left(\frac{dy}{dx}\right) \quad \text{— (i)}$$

Similarly, the excess pressure on the plane CD along the negative x axis

$$= P + \frac{dP}{dx}\delta x = -K\frac{dy}{dx} + \frac{d}{dx}\left(-K\frac{dy}{dx}\right)\delta x$$

$$= -K\frac{dy}{dx} - K\frac{d^2y}{dx^2}\delta x \quad \text{— (ii)}$$

$\therefore$ The excess pressure on the plane AB

$$= K\frac{d^2y}{dx^2}\delta x$$ (subtracting (ii) from (i))

Hence excess force on the plane AB

$$= K \frac{d^2y}{dx^2} \delta x \qquad (\because \text{unit cross-sectional area})$$

This force is impressed on the enclosed gas causing it to move. Now mass of the enclosed gas

$$= \delta x \times 1 \times \rho \qquad (\rho = \text{density of gas})$$

$\therefore$ from Newton's second law we get

$$K \frac{d^2y}{dx^2} \delta x = \delta x \times 1 \times \rho \times \frac{d^2y}{dt^2} \qquad \left(\frac{d^2y}{dt^2} = \text{accl}^n\right)$$

or
$$\frac{d^2y}{dt^2} = \frac{K}{\rho} \frac{d^2y}{dx^2} \quad \text{— (iii)}$$

This represents the D'Alembert type of wave equation. putting $\frac{K}{\rho} = v^2$ in eqn (iii) we get

$$\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2} \quad \text{— (iv)}$$

from eqn (iii) & (iv) we get

$$v^2 = \frac{K}{\rho}$$

$$\boxed{v = \sqrt{\frac{K}{\rho}}}$$

This is the required velocity of longitudinal wave in a gaseous medium.

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