Integral Domain :→

A commutative ring with unity without any divisor of zero is called integral domain.

$$\boxed{\text{Ring} + \text{Commutative} + \text{Unity} + \text{No zero divisor.}}$$

Ex Set of integers under usual addition & multiplication is an integral domain ie $(Z, +, \cdot)$ is integral domain

Ex Set of all real number under usual addition & multiplication is integral domain ie $(R, +, \cdot)$ is I.D.

Ex Let $R = \{a + b\sqrt{2} \mid a, b \in R\}$ under usual addition & multiplication is an integral domain

Field :→

A commutative ring with unity in which each non-zero element has multiplicative inverse.

$$\boxed{\text{Ring} + \text{Commutative} + \text{Unity} + \text{Non-zero element has multiplicative inv.}}$$

Ex Set of integers under usual addition & multiplication is not a field because $\exists\ 2 \in Z$ whose multiplicative inverse $\frac{1}{2} \notin Z$ $(2 \neq 0)$

Ex Set of real ~~no~~ number under usual addition and multiplication is a field. ie $(R, +, \cdot)$ is a field.

Ex Set of rational numbers under usual addition & multiplication is a field.

Ex Set of complex numbers under addition & multiplication of complex numbers is a field.

Division Ring (skew-field)

A ring with unity in which non-zero elements have multiplicative inverse is called skew field.

Ex: Let $A = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \,\middle|\, a, b \in R \right\}$

Clearly A is a ring under addition and multiplication of matrices.

Also putting $a=1$, $b=0$ we get a matrix $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \in A$ which is multiplicative identity.

$\therefore$ Let $X \in A$ $(X \neq 0)$

ie $X = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$

$|X| = a^2 + b^2 \neq 0$

$X^{-1} = \frac{1}{|X|} \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$

Hence each non-zero element has a multiplicative in A.