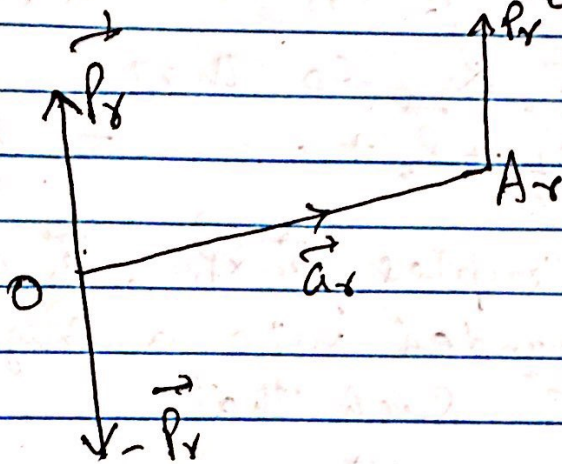


COPLANAR FORCES ACTING ON A RIGID BODY

Theorem 1: Prove that any system of coplanar forces acting upon a rigid body can be reduced to a single force acting at any arbitrarily chosen point together with a couple.
(Note: Definition of coplanar forces and its prove)

Proof:



Let P_r ($r=1, 2, 3, \dots$) be any no. of coplanar forces acting at the points A_r ($r=1, 2, 3, \dots$) of a rigid body. Let O be any point referred as origin. Let position vector of the point A_r ($r=1, 2, \dots$) be \vec{A}_r ($r=1, 2, \dots$).

We introduce two forces P_r and $-P_r$ in opposite directions and parallel to each other at O. Since P_r and $-P_r$ at O are equal in magnitudes but opposite in directions, so they balance each other and will have no effect on the system.

Now the forces P_r at A_r is equivalent to P_r at A_r , P_r at O and $-P_r$ at O.

But the forces P_r at A_r and $-P_r$ at O form a couple of moment $\vec{A}_r \times P_r$, where $r=1, 2, 3, \dots$

thus \vec{P}_1 at A_1 is equivalent to a parallel force \vec{P}_1 at O together with a couple of moment $\vec{a}_1 \times \vec{P}_1$; \vec{P}_2 at A_2 is equivalent to a parallel force \vec{P}_2 at O together with a couple of moment $\vec{a}_2 \times \vec{P}_2$ and so on.

Let the forces $\vec{P}_1, \vec{P}_2, \dots$ acting at O have resultant \vec{R} , so that

$$\vec{R} = \vec{P}_1 + \vec{P}_2 + \vec{P}_3 + \dots = \sum_{r=1} \vec{P}_r$$

Also all the couples $\vec{a}_1 \times \vec{P}_1, \vec{a}_2 \times \vec{P}_2, \dots$ can be compounded into a single couple of moment \vec{G} such that

$$\vec{G} = \vec{a}_1 \times \vec{P}_1 + \vec{a}_2 \times \vec{P}_2 + \dots = \sum_{r=1} (\vec{a}_r \times \vec{P}_r)$$

Finally we see that the given system of forces $\vec{P}_1, \vec{P}_2, \vec{P}_3, \dots$ acting at points A_1, A_2, A_3, \dots respectively is reduced to

(i) a single force \vec{R} acting at an arbitrarily chosen point O .

and (ii) a couple of moment

$$\vec{G} = \sum_{r=1} (\vec{a}_r \times \vec{P}_r)$$

This proves the theorem