$$LHS = \Delta\left(\frac{u_x}{v_x}\right) = \frac{u_{x+1}}{v_{x+1}} - \frac{u_x}{v_x}$$

$$= \frac{v_x u_{x+1} - u_x v_{x+1}}{v_x v_{x+1}}$$

$$= \frac{v_x u_{x+1} - v_x u_x + v_x u_x - u_x v_{x+1}}{v_x v_{x+1}}$$

$$= \frac{(u_{x+1} - u_x) v_x - (v_{x+1} - v_x) u_x}{v_x v_{x+1}}$$

$$= \frac{\Delta u_x v_x - \Delta v_x u_x}{v_x v_{x+1}}$$

$$= \frac{v_x \Delta u_x - u_x \Delta v_x}{v_x v_{x+1}} \quad \text{proved}$$

—x—

Q9 Explain the difference between $\left(\frac{\Delta^2}{E}\right)u_x$ and $\frac{\Delta^2 u_x}{E u_x}$ and find their values when $u_x = x^3$.

S:

(i) $\left(\frac{\Delta^2}{E}\right) u_x = \Delta^2 E^{-1} u_x$

$= \Delta^2 u_{x-1}$

$= \Delta(\Delta u_{x-1})$

$= \Delta(u_x - u_{x-1})$

$= u_{x+1} - u_x - u_x + u_{x-1}$

$= u_{x+1} - 2u_x + u_{x-1}$ —

(ii) $\frac{\Delta^2 u_x}{E u_x} = \frac{\Delta^2 u_x}{u_{x+1}} = \frac{1}{u_{x+1}} \Delta(\Delta u_x) = \frac{1}{u_{x+1}} \Delta(u_{x+1} - u_x)$

$= \frac{1}{\ } [u_{x+2} - 2u_{x+1} + u_x]$

When $u_x = x^3$

$u_{x+1} = (x+1)^3$, $u_x = x^3$, $u_{x-1} = (x-1)^3$

Putting all these values in (i) and (ii) then

$$\left(\frac{\Delta^2}{E}\right)u_x = (x+1)^3 - 2x^3 + (x-1)^3$$
$$= x^3+1+3x^2+3x-2x^3+x^3-1-3x^2+3x$$
$$= 3x+3x$$
$$= 6x$$

and

$$\frac{\Delta^2}{Eu_x}u_x = \frac{1}{(x+1)^3}\left[(x+2)^3 - 2(x+1)^3 + x^3\right]$$
$$= \frac{1}{(x+1)^3}\left[x^3+8+\cancel{6x^2}+12x-2x^3-2-\cancel{6x^2} -6x+x^3\right]$$
$$= \frac{1}{(x+1)^3}\left[6x+6\right] = \frac{1}{(x+1)^{\cancel{3}\,2}}\,6\cancel{(x+1)}$$
$$= \frac{6}{(x+1)^2}$$

—x—

(10) Explain the difference between $\left(\frac{\Delta u_x}{Eu_x}\right)^2$ and $\frac{\Delta^2}{E}u_x$ and $\left(\frac{\Delta^2 u_x}{E^2 u_x}\right)$

Ans:-

(i) $\left(\frac{\Delta u_x}{Eu_x}\right)^2 = \left(\frac{u_{x+1}-u_x}{u_{x+1}}\right)^2 = \frac{u_{x+1}^2+u_x^2-2u_{x+1}u_x}{u_{x+1}^2}$

(ii) $\left(\frac{\Delta^2}{E}\right)u_x = \Delta^2 E^{-1}u_x = \Delta^2 u_{x-1} = \Delta(\Delta u_{x-1})$
$$= \Delta(u_x - u_{x-1}) = u_{x+1}-u_x-(u_x-u_{x-1})$$
$$= u_{x+1} - 2u_x + u_{x-1}$$

(iii) $\frac{\Delta^2 u_x}{E^2 u_x} = \frac{\Delta(\Delta u_x)}{E(E u_x)} = \frac{\Delta(u_{x+1} - u_x)}{E\, u_{x+1}}$

$= \frac{u_{x+2} - u_{x+1} - (u_{x+1} - u_x)}{u_{x+2}} = \frac{u_{x+2} - 2u_{x+1} + u_x}{u_{x+2}}$

—x—

(11) Show that (i) $E^2 f(x) \neq [E f(x)]^2$ where E is an operator.

(ii) Find the function whose first difference is $e^x$.

(i) $E^2 f(x) = E E f(x)$

$= E f(x+h)$

$= f(x+2h)$ —— (1)

$[E f(x)]^2 = [f(x+h)]^2 = f^2(x+h)$ —— (2)

From (1) and (2)

$E^2 f(x) \neq [E f(x)]^2$ proved.

(ii) Let $f(x)$ be the function

Given $\Delta f(x) = e^x$

$\therefore f(x+h) - f(x) = e^x$ —— (1)

Let $f(x) = Ae^x$

$\therefore f(x+h) = A e^{x+h}$

$\therefore$ from (1)

$Ae^{x+h} - Ae^x = e^x$

$\Rightarrow A e^x (e^h - 1) = e^x$

$\Rightarrow A = \frac{e^x}{e^x (e^h - 1)}$

$$\Rightarrow A = \frac{1}{e^h - 1}$$

$$\therefore f(x) = \frac{e^x}{e^h - 1}$$

—x—

(12) Evaluate $\frac{\Delta^2 x^3}{E x^3}$

Ans:-

$$\frac{\Delta^2 x^3}{E x^3} = \frac{\Delta \Delta x^3}{(x+1)^3} = \frac{\Delta\{(x+1)^3 - x^3\}}{(x+1)^3}$$

$$= \frac{(x+2)^3 - (x+1)^3 - \{(x+1)^3 - x^3\}}{(x+1)^3}$$

$$= \frac{(x+2)^3 - 2(x+1)^3 + x^3}{(x+1)^3}$$

$$= \frac{x^3 + 8 + \cancel{6x^2} + 12x - 2x^3 - 2 - \cancel{6x^2} - 6x + x^3}{(x+1)^3}$$

$$= \frac{6x+6}{(x+1)^3} = \frac{\cancel{(x+1)}\,6}{(x+1)^{\cancel{3}\,2}} = \frac{6}{(x+1)^2}$$

—x—

Date 31/08/2018

(13) Prove that $\Delta^n x^n = \lfloor n$

Ans:-

$$\Delta x^n = (x+1)^n - x^n \quad \text{, Taking } h=1$$

$$= \left\{x^n + n x^{n-1} + \frac{n(n-1)}{\lfloor 2} x^{n-2} + \cdots + 1\right\} - x^n$$

$$= n x^{n-1} + \frac{n(n-1)}{\lfloor 2} x^{n-2} + \cdots + 1$$

$$\Delta^2 x^n = \Delta \Delta x^n$$

$$\therefore \Delta^2 x^n = \Delta\left[n x^{n-1} + \frac{n(n-1)}{\lfloor 2} x^{n-2} + \cdots + 1\right]$$

$$= n\left[(x+1)^{n-1} - x^{n-1}\right] + \frac{n(n-1)}{\lfloor 2}\left[(x+1)^{n-2} - x^{n-2}\right]$$

$$+ \cdots\cdots$$

$$= n\left[\left\{x^{n-1} + (n-1)x^{n-2} + \cdots\right\} - x^{n-1}\right] + \cdots$$

$$= n(n-1)x^{n-2} + \cdots\cdots$$

$$\Delta^3 x^n = \Delta\Delta^2 x^n$$

$$= \Delta\left[n(n-1)x^{n-2} + \cdots\right]$$

$$= n(n-1)\left[(x+1)^{n-2} - x^{n-2}\right] + \cdots$$

$$= n(n-1)\left[\left\{x^{n-2} + (n-2)x^{n-3} + \cdots\right\} - x^{n-2}\right] + \cdots$$

$$= n(n-1)(n-2)x^{n-3} + \cdots\cdots$$

Proceeding in this way we get

$$\Delta^n x^n = n(n-1)(n-2)(n-3)\cdots\cdots\{n-(n-1)\}x^{n-n}$$

$$= n(n-1)(n-2)\cdots\cdots 1$$

$$= \lfloor n \quad \text{proved}$$

Note: $\Delta^{n+1} x^n = 0$

Evaluate:

(i) $\Delta^3\left[(1-x)(1-2x)(1-3x)\right]$

(ii) $\Delta^{10}\left[(1-ax)(1-bx^2)(1-cx^3)(1-dx^4)\right]$

Ans:

(i) $\Delta^3[(1-x)(1-2x)(1-3x)]$

$= \Delta^3(-6x^3)$ {∵ The term having power than 3 will vanish

$= -6 \times \lfloor 3$

$= -6 \times 6 = -36$

(ii) $\Delta^{10}[(1-ax)(1-bx^2)(1-cx^3)(1-dx^4)]$

$= \Delta^{10}(abcd\,x^{10})$ {∵ The term having power less than 10 will be van

$= abcd\,\Delta^{10}x^{10}$

$= abcd\,\lfloor 10$ Ans

—x—

(15) Evaluate $\Delta \tan^{-1} ax$

Ans:-

∵ $\Delta f(x) = f(x+h) - f(x)$

∴ $\Delta \tan^{-1} x = \tan^{-1} a(x+1) - \tan^{-1} ax$ {Taking h=

$= \tan^{-1}\left[\frac{a(x+1) - ax}{1 + a(x+1)ax}\right]$

$= \tan^{-1}\left[\frac{a}{1 + a^2(x+1)x}\right]$ Ans

—x—

(16) Evaluate

a) $\Delta^n(ab^{cx})$

Ans:

$\Delta(ab^{cx}) = ab^{c(x+1)} - ab^{cx}$

$= (b^c - 1)ab^{cx}$

Again $\Delta^2(ab^{cx}) = \Delta\{(b^c-1)ab^{cx}\}$

$$\Delta^2(ab^{cx}) = (b^c-1)\,\Delta(ab^{cx})$$
$$= (b^c-1)(b^c-1)\,ab^{cx}$$
$$= (b^c-1)^2\,ab^{cx}$$

Proceeding in this way, we get

$$\Delta^n(ab^{cx}) = (b^c-1)^n\,ab^{cx}$$

3 (b)

$$\Delta^2\left[\frac{5x+12}{x^2+5x+6}\right]$$

Sol:

$$\Delta^2\left[\frac{5x+12}{x^2+5x+6}\right] = \Delta\left[\frac{5(x+1)+12}{(x+1)^2+5(x+1)+6} - \frac{5x+12}{x^2+5x+6}\right]$$

$$= \Delta\left[\frac{5x+17}{x^2+2x+1+5x+5+6} - \frac{5x+12}{x^2+5x+6}\right]$$

$$= \Delta\left[\frac{5x+17}{x^2+7x+12} - \frac{5x+12}{x^2+5x+6}\right]$$

$$= \left[\frac{5(x+1)+17}{(x+1)^2+7(x+1)+12} - \frac{5x+17}{x^2+7x+12}\right]$$

$$- \left[\frac{5(x+1)+12}{(x+1)^2+5(x+1)+6} - \frac{5x+12}{x^2+5x+6}\right]$$

$$= \left[\frac{5x+22}{x^2+9x+20} - \frac{5x+17}{x^2+7x+12}\right] - \left[\frac{5x+17}{x^2+7x+12} - \frac{5x+12}{x^2+5x+6}\right]$$

$$= \frac{5x+22}{x^2+9x+20} - \frac{10x+34}{x^2+7x+12} + \frac{5x+12}{x^2+5x+6}$$

Ans