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Important Notes:

Note 1: Any system of coplanar forces acting on a rigid body can be reduced ultimately to either a single force or a single couple, unless it is in equilibrium.

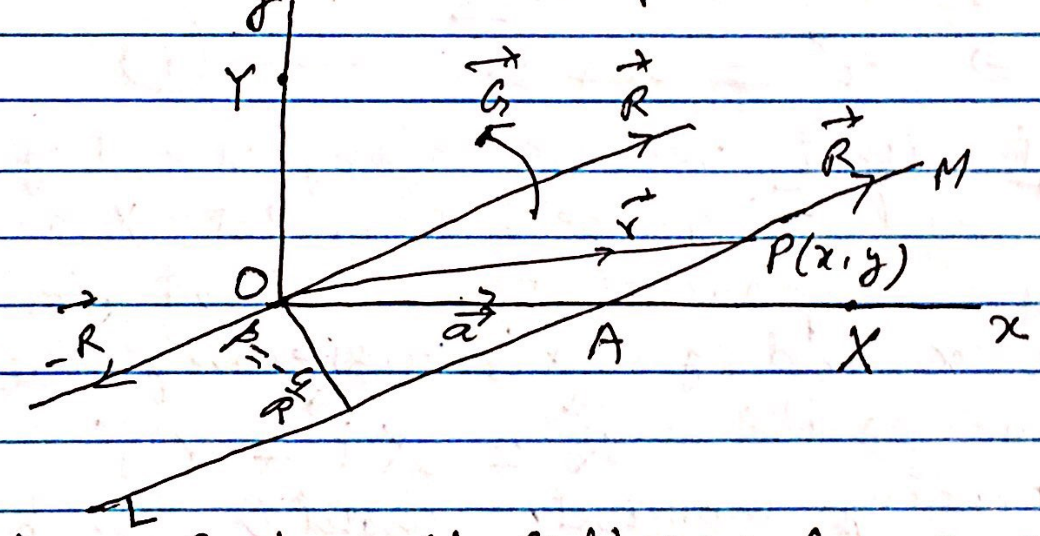
Note 2: Resultant of forces $\vec{R}$ does not change with the change of origin O.

Note 3: The moment $\vec{G}$ depends upon the choice of origin.

Theorem 2: Equation of the line of action of the resultant.

Statement: Find the equation of the line of action of the resultant of a system of coplanar forces acting upon a rigid body.

Proof:



Let given system of coplanar forces reduced to a single force $\vec{R}$ at the origin O together with a couple of moment $\vec{G}$.

The couple of moment $\vec{G}$ can be replaced by two equal and unlike parallel forces $-\vec{R}$ at O and $\vec{R}$ at a distance $\frac{G}{R}$ from O

and parallel to the force $\vec{R}$ at O. The forces $\vec{R}$ and $-\vec{R}$ at O balance each other and then given system of forces reduced to a single resultant force $\vec{R}$ along a line LM at a distance $\frac{G}{R}$ from O and parallel to the original force at O.

Let LM cuts the x-axis at the point A whose position vector is $\vec{a}$. Let P be any point on LM, whose position vector is $\vec{r}$.

Since $\vec{AP}$ is parallel to $\vec{R}$, Then

$\vec{AP} = s\vec{R}$, where s is scalar.

$\Rightarrow \vec{OP} - \vec{OA} = s\vec{R}$, where O is the origin.

$\Rightarrow \vec{r} - \vec{a} = s\vec{R}$

$\Rightarrow \vec{r} = \vec{a} + s\vec{R}$ ——— (1)

Let unit vectors along x and y axes are $\vec{i}$ and $\vec{j}$ respectively. Let X and Y are the components of resultant force $\vec{R}$, parallel to x and y axes respectively. Then we can write $\vec{R} = X\vec{i} + Y\vec{j}$

Taking moments about O, we have

$$aY = G \Rightarrow a = \frac{G}{Y}$$

Now by (1), $\vec{r} = \left(\frac{G}{Y}\right)\vec{i} + s\vec{R}$ $(\because \vec{a} = a\vec{i})$

Thus the vector equation of the line of action LM of $\vec{R}$ is

$$\boxed{\vec{r} = \left(\frac{G}{Y}\right)\vec{i} + s\vec{R}.}$$

Cartesian equation of line of action

$\because$ vector equation of the line of action is

$$\vec{r} = \frac{G}{Y}\vec{i} + s\vec{R}$$, Then we may write-

$$x\vec{i} + y\vec{j} = \frac{G}{Y}\vec{i} + s(X\vec{i} + Y\vec{j})$$

where $\vec{r} = x\vec{i} + y\vec{j}$

Equating the coefficients of $\vec{i}$ and $\vec{j}$ on both sides, we have

$$x = \frac{G}{Y} + sX \quad \text{and} \quad y = sY$$

Eliminiting s, we have

$$x = \frac{G}{Y} + \frac{y}{Y}X \Rightarrow xY = G + yX$$

$$\Rightarrow \boxed{xY - yX = G}$$

This is the equation of the line of action of the forces in the Cartesian form.

$\times$

Some important Remarks:

1. If $\vec{R} = 0$ and $\vec{G} \neq 0$, then all the given forces reduce to a single Couple of moment with magnitude G and the body has only a motion of rotation $\circlearrowleft$ or $\circlearrowright$.

2. If $\vec{R} \neq 0$ and $\vec{G} = 0$, then all the given forces reduce to a single resultant force $\vec{R}$ and the body has only a motion of translation ($\longrightarrow$). In this case the sum of all moments about any point is zero.

3. If $\vec{R} \neq 0$ and $\vec{G} \neq 0$, then the body has in both the motions i.e in the motion of translation and the motion of rotation. In this case $\vec{R}$ and $\vec{G}$ can be compounded into a single resultant force $\vec{R}$ parallel to the original force $\vec{R}$ and a distance $\frac{G}{R}$ from it.

4. <u>Necessary Conditions for equilibrium:</u>

Let the given forces are in equilibrium. This means that the body has neither the motion of translation nor the motion of rotation.

Then $\vec{R} = 0$ and $\vec{G} = 0$

$$\Rightarrow \sum_{r=1} \vec{P_r} = 0 \text{ and } \sum_{r=1} (\vec{a_r} \times \vec{P_r}) = 0$$

These are the necessary conditions of equilibrium.

<u>Sufficient conditions for equilibrium:</u>

Let $\vec{R} = 0$ and $\vec{G} = 0$. Then

$\vec{R} = 0 \Rightarrow$ there is not translational motion

$\vec{G} = 0 \Rightarrow$ there is no rotational motion in the body.

Hence there is no kind of displacement of the body is possible and so the forces are in equilibrium.

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