

Theorem 3: Find the necessary and sufficient for a system of coplanar forces to be in equilibrium.

Proof: let $\vec{F}_1, \vec{F}_2, \vec{F}_3, \dots$ be a given system of coplanar forces acting at the points A_1, A_2, A_3, \dots of a rigid body. Taking arbitrary point "O" as origin. let the position vectors of the points A_1, A_2, A_3, \dots are $\vec{a}_1, \vec{a}_2, \vec{a}_3, \dots$ respectively.

Thus we know that these forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, \dots$ can be reduced to a single force \vec{R} at O together with a couple \vec{G} such that

$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots = \sum \vec{F}_i$$

$$\text{and } \vec{G} = \vec{a}_1 \times \vec{F}_1 + \vec{a}_2 \times \vec{F}_2 + \vec{a}_3 \times \vec{F}_3 + \dots = \sum_{i=1}^n \vec{a}_i \times \vec{F}_i$$

Where \vec{R} is the sum of all forces shifted at O and \vec{G} is the sum of the moments of all forces about O.

Necessary Condition: let the given forces are in equilibrium.

It means that the body has neither in the motion of translation nor in the motion of the rotation.

$$\text{i.e. } \vec{R} = \vec{0} \quad \text{and} \quad \vec{G} = \vec{0}$$

$$\Rightarrow \sum_{i=1}^n \vec{F}_i = \vec{0} \quad \text{and} \quad \sum_{i=1}^n \vec{a}_i \times \vec{F}_i = \vec{0}$$

These are necessary conditions of equilibrium

Sufficient Condition: let $\vec{R} = 0$ and $\vec{G} = 0$
that means

$\vec{R} = 0 \rightarrow$ there is no translational motion in the body.

and $\vec{G} = 0 \rightarrow$ there is no rotational motion.

Hence under the assumption, there is no kind of displacement of the body took place. So the forces are in equilibrium.

Thus $\sum_{r=1}^n \vec{P}_r = 0$ and $\sum (\vec{a}_r \times \vec{P}_r) = \vec{0}$ are

the set of n & s conditions of equilibrium of the body.

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