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$\therefore$ n^{th} difference is

$$\Delta^n e^{a+bx} = (e^b - 1)^n e^{a+bx}$$

(d) $\Delta(x!)$

Ans: $\Delta(\underline{|x}) = \underline{|x+1} - \underline{|x}$
$= \underline{|x}\,(x+\cancel{1}-\cancel{1})$
$= x\,\underline{|x}$

(e) $\Delta^n(ax^n + bx^{n-1})$.

Ans: $\Delta^n(ax^n + bx^{n-1}) = a\Delta^n x^n + b\Delta^n x^{n-1}$
$= a\,\underline{|n} + b \cdot 0$ $\left\{ \because \Delta^n x^n = \underline{|n}\right.$ the terms of Power less than will be 0
$= a\,\underline{|n}$

f) $\Delta^n\left(\frac{1}{x}\right)$

Ans: $\Delta\left(\frac{1}{x}\right) = \frac{1}{x+1} - \frac{1}{x}$
$= \frac{\cancel{x} - \cancel{x} - 1}{x(x+1)}$
$= \frac{-1}{x(x+1)}$

11. Show that $\Delta^3 y_i = y_{i+3} - 3y_{i+2} + 3y_{i+1} - y_i$

Ans:
$$\Delta^3 y_i = \Delta^2(\Delta y_i)$$
$$= \Delta^2(y_{i+1} - y_i)$$
$$= \Delta\{\Delta y_{i+1} - \Delta y_i\}$$
$$= \Delta\{(y_{i+2} - y_{i+1}) - (y_{i+1} - y_i)\}$$
$$= \Delta\{y_{i+2} - 2y_{i+1} + y_i\}$$
$$= \Delta y_{i+2} - 2\Delta y_{i+1} + \Delta y_i$$
$$= (y_{i+3} - y_{i+2}) - 2(y_{i+2} - y_{i+1}) + (y_{i+1} - y_i)$$
$$= y_{i+3} - 3y_{i+2} + 3y_{i+1} - y_i$$

Proved

12. Prove that

(i) $y_3 = y_2 + \Delta y_1 + \Delta^2 y_0 + \Delta^3 y_0$

Ans. R.H.S

$$y_2 + \Delta y_1 + \Delta^2 y_0 + \Delta^3 y_0$$
$$= y_2 + (y_2 - y_1) + \Delta^2 y_0 + \Delta^2(y_1 - y_0)$$
$$= y_2 + y_2 - y_1 + \cancel{\Delta^2 y_0} + \Delta^2 y_1 - \cancel{\Delta^2 y_0}$$
$$= 2y_2 - y_1 + \Delta(y_2 - y_1)$$
$$= 2y_2 - y_1 + \Delta y_2 - \Delta y_1$$
$$= 2y_2 - y_1 + (y_3 - y_2) - (y_2 - y_1)$$
$$= 2\cancel{y_2} - \cancel{y_1} + y_3 - \cancel{y_2} - \cancel{y_2} + \cancel{y_1}$$
$$= y_3 = \text{L.H.S.}$$

Proved

$$\Delta^2\left(\frac{1}{x}\right) = \Delta\left\{\frac{-1}{x(x+1)}\right\}$$

$$= \frac{-1}{(x+1)(x+2)} + \frac{1}{x(x+1)}$$

$$= \frac{-\cancel{x} + \cancel{x} + 2}{x(x+1)(x+2)}$$

$$= \frac{2}{x(x+1)(x+2)}$$

$$\Delta^3\left(\frac{1}{x}\right) = \Delta\left\{\Delta^2\left(\frac{1}{x}\right)\right\}$$

$$= \Delta\left\{\frac{2}{x(x+1)(x+2)}\right\}$$

$$= \frac{2}{(x+1)(x+2)(x+3)} - \frac{2}{x(x+1)(x+2)}$$

$$= \frac{\cancel{2x} - \cancel{2x} - 6}{x(x+1)(x+2)(x+3)}$$

$$= \frac{-1\cdot 2\cdot 3}{x(x+1)(x+2)(x+3)}$$

$\therefore$ n^{th} difference is

$$\Delta^n\left(\frac{1}{x}\right) = \frac{(-1)^n \, \underline{|n}}{x(x+1)(x+2)\cdots(x+n)}$$

(ii) $\nabla^2 y_8 = y_8 - 2y_7 + y_6$

Ans L.H.S

$$\nabla^2 y_8 = \nabla(\nabla y_8)$$
$$= \nabla(y_8 - y_7)$$
$$= \nabla y_8 - \nabla y_7$$
$$= y_8 - y_7 - (y_7 - y_6)$$
$$= y_8 - y_7 - y_7 + y_6$$
$$= y_8 - 2y_7 + y_6 = R.H.S$$

Proved

3. Evaluate

i) $\Delta x(x+1)(x+2)(x+3)$

$$\Delta x(x+1)(x+2)(x+3)$$
$$= (x+1)(x+2)(x+3)(x+4) - x(x+1)(x+2)(x+3)$$
$$= (x+1)(x+2)(x+3)\{\cancel{x}+4-\cancel{x}\}$$
$$= 4(x+1)(x+2)(x+3) \quad A$$

$\Delta^n(x^n)$

$$\Delta^n x^n = \Delta^{n-1}\{\Delta x^n\}$$
$$= \Delta^{n-1}\{(x+1)^n - x^n\}$$
$$= \Delta^{n-1}\left[\cancel{x^n} + {}^nC_1 x^{n-1} + {}^nC_2 x^{n-2} + \cdots \right.$$
$$\left. \cdots x^n\right]$$

$= \Delta^{n-1} \{ {}^{n}C_1 x^{n-1} \}$
$= \Delta^{n-1} (n x^{n-1})$
$= n \Delta^{n-2} (\Delta x^{n-1})$
$= n \Delta^{n-2} \{ (x+1)^{n-1} - x^{n-1} \}$
$= n \, {}^{n-1}C_1 \Delta^{n-2} (x^{n-2})$ {as above}
$= n(n-1) \Delta^{n-2} (x^{n-2})$
$= n(n-1) \cdots 3.2. \Delta(x)$
$= n(n-1) \cdots 3.2.1$
$= \lfloor n$ as $\Delta x = (x+1) - x$
$= 1$

Note: $\delta^{n+1} x^n = 0$

1\. Evaluate

$\Delta^3 [(1-x)(1-2x)(1-3x)]$

$\Delta^3 [(1-x)(1-2x)(1-3x)]$
$= \Delta^3 [-x \cdot (-2x) \cdot (-3x)]$
$= \Delta^3 (-6x^3)$ {$\because$ The terms having Power less than 3 vanish}

Now,
$\Delta^3 (-6x^3) = -6 \Delta^3 x^3$ {$\because \Delta^n x^n = \lfloor n$}
$= -6 \lfloor 3$
$= -36$

$\Delta^{10}[(1-ax)(1-bx^2)(1-cx^3)(1-dx^4)]$

$\Delta^{10}[(1-ax)(1-bx^2)(1-cx^3)(1-dx^4)]$

$= \Delta^{10}[(-ax)(-bx^2)(-cx^3)(-dx^4)]$

$\Delta^{10}[abcd\,x^{10}]$ { ∵ the terms of Power less than 10 will be vanish }

ow,

$\Delta^{10}(abcd\,x^{10}) = abcd\,\Delta^{10}x^{10}$

$= abcd\,\lfloor 10$ { $\because \Delta^n x^n = \lfloor n$ }

rove that

$(4) = f(3) + \Delta f(2) + \Delta^2 f(1) + \Delta^3 f(1)$

$\because \Delta f(3) = f(4) - f(3)$

$$
\begin{aligned}
f(4) &= f(3) + \Delta f(3) \\
&= f(3) + \Delta\{f(2) + \Delta f(2)\} \\
&= f(3) + \Delta f(2) + \Delta^2 f(2) \\
&= f(3) + \Delta f(2) + \Delta^2\{f(1) + \Delta f(1)\} \\
&= f(3) + \Delta f(2) + \Delta^2 f(1) + \Delta^3 f(1)
\end{aligned}
$$

Proved

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
0	y_0					
		$y_1 - y_0 = \Delta y_0$				
1	y_1		$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$			
		$y_2 - y_1 = \Delta y_1$		$\Delta^3 y_0$		
2	y_2		$\Delta^2 y_1$		$\Delta^4 y_0$	
		$y_3 - y_2 = \Delta y_2$		$\Delta^3 y_1$		$\Delta^5 y_0$
3	y_3		$\Delta^2 y_2$		$\Delta^4 y_1$	
		$y_4 - y_3 = \Delta y_3$		$\Delta^3 y_2$		
4	y_4		$\Delta^2 y_3$			
		$y_5 - y_4 = \Delta y_4$				
5	y_5					

If the following table is given for the function $y = f(x)$, then form the diagonal table:

x: 1 2 3 4 5 6 7
y: 0 1 16 81 256 625 1296

Also read $\Delta^2 y_3$, $\Delta^3 y_2$, $\Delta^4 y_1$

The diagonal difference table is given below -

$= \cancel{f(1) + 3f(2) - 3f(4) + 3f(3) - 6f(2) + 3f(1) + \Delta^3 f(1)}$

$\cancel{= f(3) + [f(3) - f(2)] + [f(3) - 2f(2) + f(1)]}$

(ii) $f(4) = f(0) + 4\Delta f(0) + 6\Delta^2(-1) + 10\Delta^3 f(-1)$

as far as 3rd difference

Sol: $f(4) = E^5 f(-1)$

$= (1+\Delta)^5 f(-1)$

$= (1 + {}^5C_1\Delta + {}^5C_2\Delta^2 + {}^5C_3\Delta^3 + {}^5C_4\Delta^4 + {}^5C_5\Delta^5) f(-1)$

$= f(-1) + 5\Delta f(-1) + 10\Delta^2 f(-1) + 10\Delta^3 f(-1) + 5\Delta^4 f(-1) + \Delta^5 f(-1)$

$= [f(-1) + \Delta f(-1)] + 4\Delta f(-1) + 10\Delta^2 f(-1) + 10\Delta^3 f(-1)$

as given 3rd diff

$= [\cancel{f(-1)} + f(0) - \cancel{f(-1)}] + 4\Delta f(-1) + 4\Delta^2 f(-1) + 6\Delta^2 f(-1) + 10\Delta^3 f(-1)$

$= f(0) + 4\Delta[f(-1) + \Delta f(-1)] + 6\Delta^2 f(-1) + 10\Delta^3 f(-1)$

$= f(0) + 4\Delta f(0) + 6\Delta^2 f(-1) + 10\Delta^3 f(-1)$

NOTE: we consider the following table

x	x_0	x_1	x_2	x_3	x_4	x_5
y	y_0	y_1	y_2	y_3	y_4	y_5