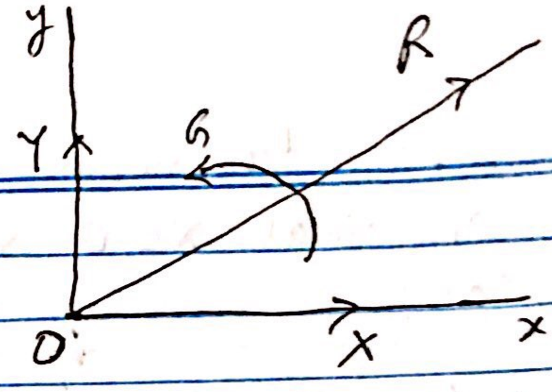
— x —

Theorem 4 Prove that a n & s condition of equilibrium of a system of coplanar forces acting on a rigid body are that
(1) The algebraic sum of the resolved parts of all forces in any two perpendicular directions should be separately zero, and
(2) The algebraic sum of the magnitudes of the moments of all the forces about any point in the plane should be zero.

Proof: Let G be the sum of the moments of all the forces about O and R is the sum of all forces, where O is the arbitrary choosen point.



Since we know that the given system of forces can be reduced to a single force R acting at an arbitrarily chosen point O, together with a couple.

Through O, we draw two mutually perpendicular lines Ox and Oy, treating as coordinate axes.

Let X and Y be the algebraic sums of the resolved parts of all the forces in the directions of Ox and Oy respectively. Then X and Y are the resolved parts of resultant force R in these directions.

Hence $R = \sqrt{X^2+Y^2}$

Then the given system of forces is equivalent to the forces X, Y and a couple of moment G.

Necessary Condition let us assume the given forces are in equilibrium.

Since a force and a couple of moment can not balance one another, then it is necessary that R and G should be separately zero.

$\therefore \; R = 0$ and $G = 0$

But $R = \sqrt{X^2+Y^2}$

$\therefore \; R = 0 \Leftrightarrow X = 0, \; Y = 0.$

Hence the necessary conditions for equilibrium are that $X=0$, $Y=0$ and $G=0$

Sufficient conditions:

Let $X=0$, $Y=0$ and $G=0$

Now we shall show that the given system of forces is in equilibrium.

$\because$ $X=0, Y=0 \Rightarrow R=\sqrt{X^2+Y^2}=0$

$\Rightarrow$ The body has no motion of translation.

Again $G=0 \Rightarrow$ The algebraic sum of moments of all the forces about any given point is zero.

$\Rightarrow$ The body has no motion of rotation.

Thus no kind of displacement of the body is possible and the forces are in equilibrium. Hence the given conditions for equilibrium are sufficient.

Hence given conditions for equilibrium are

$$\left.\begin{array}{r} R=0 \\ \text{and } G=0. \end{array}\right\} \text{ or } \begin{array}{c} X=0,\ Y=0 \\ \text{and } G=0. \end{array}$$