x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1	0					
		$1 = \Delta y_1$				
2	1		$14 = \Delta^2 y_1$			
		$15 = \Delta y_2$		$36 = \Delta^3 y_1$		
3	16		$50 = \Delta^2 y_2$		$24 = \Delta^3 y_1$	
		$65 = \Delta y_3$		$60 = \Delta^3 y_2$		0
4	81		$110 = \Delta^2 y_3$		24	
		$175 = \Delta y_4$		84		0
5	256		194		24	
		$369 = \Delta y_5$		108		
6	625		302			
		$671 = \Delta y_6$				
7	1296					

$\therefore \Delta^2 y_3 = 110, \quad \Delta^3 y_2 = 60, \quad \Delta^3 y_1 = 24$

17. Construct the table of difference for the data below.

x : 0 1 2 3 4

$f(x)$: 1.0 1.5 2.2 3.1 4.6

Evaluate $\Delta^3 f(1)$.

Ans: The table of difference given below:-

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0	1.0				
		0.5			
1	1.5		0.2		
		0.7		0	
2	2.2		0.2		0.4
		0.9		0.4	
3	3.1		0.6		
		1.5			
4	4.6				

$$\Delta^3 f(1) = 0.4$$

18) If $u_0 = 3$, $u_1 = 12$, $u_2 = 81$, $u_3 = 2000$, $u_4 = 100$
Calculate $\Delta^4 u_0$.

Ans: The difference table is given below:

x	u_x	Δu_x	$\Delta^2 u_x$	$\Delta^3 u_x$	$\Delta^4 u_x$
0	3				
		9			
1	12		60		
		69		1790	
2	81		1850		-7459
		1919		-5669	
3	2000		-3819		
		-1900			
4	100				

$\therefore \ \Delta^4 U_0 = -7459$

Remember :- If no. of entries = n (given) then Polynomial is of $(n-1)^{th}$ degree

$\therefore \ \Delta^{n-1} f(x) = \text{Const};$

and $\Delta^n f(x) = 0.$

19) Form the table of backward difference of the function $f(x) = x^3 - 3x^2 + 5x - 7$ for $x = -1, 0, 1, 2, 3, 4, 5$. Also find the values of $\nabla^2 f(3)$, $\nabla^3 f(5)$.

Ans: The table of backward difference given below.

x	$f(x)$	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$	$\nabla^4 f(x)$
-1	-6				
		$-1 = \nabla f(0)$			
0	-7		$-6 = \nabla^2 f(1)$		
		$-7 = \nabla f(1)$		$6 = \nabla^3 f(2)$	
1	-14		$0 = \nabla^2 f(2)$		$0 = \nabla^4 f(3)$
		$-7 = \nabla f(2)$		$6 = \nabla^3 f(3)$	
2	-21		$6 = \nabla^2 f(3)$		$0 = \nabla^4 f(4)$
		$-1 = \nabla f(3)$		$6 = \nabla^3 f(4)$	
3	-22		$12 = \nabla^2 f(4)$		$0 = \nabla^4 f(5)$
		$11 = \nabla f(4)$		$6 = \nabla^3 f(5)$	
4	-11		$18 = \nabla^2 f(5)$		
		$29 = \nabla f(5)$			
[illegible]	18				

So Putting $\Delta^3 f(x) = 0$ for every values of x, we get

$$x - 30 = 0$$

i.e; $\boxed{x = 30}$

and $y - 3x + 48 = 0$

$$y = 3x - 48$$

$$y = 3 \times 30 - 48$$

$$\boxed{y = 42}$$

Thus 7^{th} and 8^{th} values are 30 and 42 resp. It is 2nd degree Polynomial since 2nd difference is Const.

1) Verify the value of y_x when $x = 10$ is 920, if $y_x = x^3 - x^2 + x + 10$ and only 6 values of y_x corresponding as $x = 1, 2, 3, 4, 5, 6$ are used. Also prove that the second diff. is $6x + 4$, verify that numerically for the known part of the series.

Since $y_x = x^3 - x^2 + x + 10$ and taking A, B, C and D as 7^{th}, 8^{th}, 9^{th} and 10^{th} entry values respectively the following difference table is obtained.

$\therefore \; \nabla^2 f(3) = 6$

and $\nabla^3 f(5) = 6$

20) By constructing difference table find the 7^{th} and 8^{th} terms of the series 0, 0, 2, 6, 12, 20. Discuss the degree of polynomial.

Ans: Suppose 7^{th} and 8^{th} entry values of the series are x and y respectively. Then the following difference table is obtained.

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
1	0			
		0		
2	0		2	
		2		0
3	2		2	
		4		0
4	6		2	
		6		0
5	12		2	
		8		$x-30$
6	20		$x-28$	
		$x-20$		$y-3x+48$
7	x		$y-2x+20$	
		$y-x$		
8	y			

and $D - 3C + 3B - A = 0$

$\Rightarrow D = 800$

$\therefore f(x) = 800$, when $x = 10$ is verified.

2nd Part: $\Delta y_x = y_{x+1} - y_x$

$= [(x+1)^3 - (x+1)^2 + (x+1) + 10] - [x^3 - x^2 + x + 10]$

$= 3x^2 + x + 1$

$\Delta^2 y_x = [3(x+1)^2 + (x+1) + 1] - [3x^2 + x + 1]$

$= 6x + 4$

Verification: $\therefore \Delta^2 y_x = 6x + 4$

$\Delta^2 y_1 = 10$, $\Delta^2 y_2 = 6 \times 2 + 4 = 16$

$\Delta^2 y_3 = 6 \times 3 + 4 = 22$, $\Delta^2 y_4 = 6 \times 4 + 4 = 28$

These values are same as obtained in difference table.

Find the missing values in the following ta

x:	45	50	55	60	65
y:	3.0	—	2.0	—	-2.4

Taking y_1, y_3 as 2nd & 4th entry values, the following difference table is obtained

x	y_x	Δy_x	$\Delta^2 y_x$	$\Delta^3 y_x$
1	11			
		5		
2	16		10	
		15		6
3	31		16	
		31		6
4	62		22	
		53		6
5	115		28	
		81		$A-305$
6	196		$A-277$	
		$A-196$		$B-3A+473$
7	A		$B-2A+196$	
		$B-A$		$C-3B+3A-196$
8	B		$C-2B+A$	
		$C-B$		$D-3C+3B-A$
9	C		$D-2C+B$	
		$D-C$		
10	D			

Third difference are constant

$\therefore \quad A-305=0 \Rightarrow A=305$

and $\quad B-3A+473=0$

$\Rightarrow B=442$

and $\quad C-3B+3A-196=0$

$\Rightarrow C=607$

x	y	Δy_x	$\Delta^2 y_x$	$\Delta^3 y_x$
45	3.0			
		$y_1 - 3$		
50	y_1		$5 - 2y_1$	
		$2 - y_1$		$y_3 + 3y_1 - 9$
55	2.0		$y_3 + y_1 - 4$	
		$y_3 - 2$		$3.6 - 3y_3 - y_1$
60	y_3		$-0.4 - 2y_3$	
		$-2.4 - y_3$		
65	-2.4			

As only three entries y_0, y_2, y_4 are given then function y can be represented by a second degree polynomial.

$\therefore \Delta^3 y_0 = 0$ and $\Delta^3 y_1 = 0$

i.e; $3y_1 + y_3 = 9$ and $3.6 - 3y_3 - y_1$

$y_1 + 3y_3 = 3.6$

Solving there we get

$3y_1 + y_3 = 9 \times 1$

$y_1 + 3y_3 = 3.6 \times 3$

$$\begin{aligned} \cancel{3y_1} + y_3 &= 9 \\ -\cancel{3y_1} \pm 9y_3 &= \pm 10.8 \\ \hline -8y_3 &= -1.8 \\ \Rightarrow y_3 &= 0.225 \end{aligned}$$

$\therefore 3y_1 + y_3 = 9$

$\Rightarrow 3y_1 + 0.225 = 9$

$\Rightarrow 3y_1 = 9 - 0.225$