29) Estimate the missing term in the following table:

x :	0	1	2	3	4
$f(x)$:	1	3	9	—	81

Ans: Taking y_3 as 4th entry values the following difference table is obtained

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0	1				
		2			
1	3		4		
		6		$y_3 - 10$	
2	9		$y_3 - 15$		$124 - 4y_3$
		$y_3 - 9$		$105 - 3y_3$	
3	y_3		$90 - 2y_3$		
		$81 - y_3$			
4	81				

As only four entries are given, so the functio can be 3rd degree polynomial.

$$\therefore \Delta^4 f(x) = 0$$

$$124 - 4y_3 = 0$$

$$y_3 = \frac{124}{4} \quad 31$$

$$\therefore y_3 = 31$$

24) find the missing values in the following table:

x:	0	5	10	15	20	25
y:	6	10	—	17	—	31

Ans: Taking y_3, y_5 as 3rd and 5th entry values the difference table is given below.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	6				
		4			
5	10		$y_3 - 14$		
		$y_3 - 10$		$41 - 3y_3$	
10	y_3		$27 - 2y_3$		$6y_3 + y_5 - 102$
		$17 - y_3$		$3y_3 + y_5 - 61$	
15	17		$y_3 + y_5 - 34$		$143 - 4y_3 - 4y_5$
		$y_5 - 17$		$82 - y_3 - 3y_5$	
20	y_5		$48 - 2y_5$		
		$31 - y_5$			
25	31				

Since only 4 entries are given, So the function is of 3rd degree Polynomial.

$\therefore \quad \Delta^4 y_0 = 0$ and $\Delta^4 y_1 = 0$

$6y_3 + y_5 = 102$ and $4y_3 + 4y_5 = 143$

Solving them

$$y_3 = \frac{265}{20} = 13.25$$

$$y_5 = 102 - 79.50 = 22.50$$

$\therefore y_3 = 13.25$ and $y_5 = 22.50$

25) find the first term of the series whose second and subsequent term are 8, 3, 0, -1, 0

Ans: let first term is 'a' the difference table is

x	f(x)	Δf(x)	$\Delta^2 f(x)$	$\Delta^3 f(x)$
1	a			
		8-a		
2	8		a-13	
		-5		15-a
3	3		2	
		-3		0
4	0		2	
		-1		0
5	-1		2	
		1		
6	0			

from table,

$\because 15 - a = 0$

$a = 15$

$\therefore$ first term is 15.

26) find the six term of the sequence 6, 11, 18, 27 Also find the general term. (or find the Polynomial).

Ans The divided difference table is

x	y_x	Δy_x	$\Delta^2 y_x$	$\Delta^3 y_x$
1	6			
		5		
2	11		2	
		7		0
3	18		2	
		9		0
4	27		2	
		11		
5	38			

∵ 3rd divided difference 0, So it is a 2nd degree Polymomial.

let y_6 = 6th term

$= E^5 y_1$

$= (1+\Delta)^5 y_1$

$= \left\{1 + 5\Delta + \frac{5(5-1)}{\lfloor 2}\Delta^2 + \frac{5(5-1)(5-2)}{\lfloor 3}\Delta^3 + \cdots\right\} y$

$= y_1 + 5\Delta y_1 + \frac{5\times 4}{2}\Delta^2 y_1 + \frac{5\times 4\times 3}{\lfloor 3}\Delta^3 y_1 + \cdots$

$= 6 + 5\times 5 + \frac{5\times 4}{2}\times 2 + 0 + 0$

$= 6 + 25 + 20$

$= 51$

2nd Part :

let y_x = x^{th} term = general term

$$= E^{x-1} y_1$$

$$= (1+\Delta)^{x-1} y_1$$

$$= \left\{1 + (x-1)\Delta + \frac{(x-1)(x-2)}{\lfloor 2}\Delta^2 + \frac{(x-1)(x-2)(x-3)}{\lfloor 3}\Delta^3 + \cdots\right\} y_1$$

$$= y_1 + (x-1)\Delta y_1 + \frac{(x-1)(x-2)}{\lfloor 2}\Delta^2 y_1 + \frac{(x-1)(x-2)(x-3)}{\lfloor 3}\Delta^3 y_1 + \cdots$$

$$= 6 + (x-1)\times 5 + \frac{(x-1)(x-2)\times 2}{2} + 0 + 0 \quad \because$$

$$= 6 + 5x - 5 + x^2 - 2x - x + 2$$

$$= x^2 + 2x + 3$$

27) find the 7^{th} term of the Sequence 2, 9, 28, 65, 126, 21 and find the Polynomial.

Ans The divided difference table is given below-

x	y_x	Δy_x	$\Delta^2 y_x$	$\Delta^3 y_x$	$\Delta^4 y_x$
1	2				
		7			
2	9		12		
		19		6	
3	28		18		0
		37		6	
4	65		24		0
		61		6	
5	126		30		
		91			
6	217				

$\because$ 4th divided diff. 0, So it is 3rd degree Polymomial.

let y_7 = 7th term

$= E^6 y_1$

$= (1+\Delta)^6 y_1$

$= \left\{1 + 6\Delta + \frac{6(6-1)}{\underline{2}}\Delta^2 + \frac{6(6-1)(6-2)}{\underline{3}}\Delta^3 + \cdots\right\} y_1$

$= y_1 + 6\Delta y_1 + \frac{6\times 5}{2\times 1}\Delta^2 y_1 + \frac{6\times 5\times 4}{3\times 2\times 1}\Delta^3 y_1 + \cdots$

$= 2 + 6\times 7 + 15\times 12 + 20\times 6 + 0 + 0 + \cdots$

$= 2 + 42 + 180 + 120$

$= 344$

2nd Part:

let y_x = x^{th} term = general term

$= E^{x-1} y_1$

$= (1+\Delta)^{x-1} y_1$

$= \left\{1 + (x-1)\Delta + \frac{(x-1)(x-2)}{\lfloor 2}\Delta^2 + \frac{(x-1)(x-2)(x-3)}{\lfloor 3}\Delta^3 + \cdots\right\} y_1$

$= y_1 + (x-1)\Delta y_1 + \frac{(x-1)(x-2)}{2}\Delta^2 y_1 + \frac{(x-1)(x-2)(x-3)}{\lfloor 3}\Delta^3 y_1 + \cdots$

$= 2 + (x-1)\times 7 + \frac{(x-1)(x-2)}{2}\times 12 + \frac{(x-1)(x-2)(x-3)}{3\times 2}\times 6$

$= 2 + 7x - 7 + (x^2 - 3x + 2)6 + (x^2 - 3x + 2)(x-3)$

$= 2 + 7x - 7 + 6x^2 - 18x + 12 + x^3 - 3x^2 + 2x - 3x^2 + 9x - 6$

$= x^3 + 3x^2 - 3x^2 - 9x + 9x + 6 - 5$

$= x^3 + 1$ ∧

28) find the y_x if $y_0 = 9$, $y_1 = 18$, $y_2 = 20$, $y_3 = 24$ the 3rd difference Constant.

Ans: The ~~divided~~ difference table is given below:- ∧

x	y_x	Δy_x	$\Delta^2 y_x$	$\Delta^3 y_x$
0	9			
		9		
1	18		-7	
		2		9
2	20		2	
		4		
3	24			

Since 3rd difference is Constant, So it is a 3rd degree Polymomial.

let y_6 = 6th term

$= E^6 y_0$

$= (1+\Delta)^6 y_0$

$= \{1 + 6\Delta + \frac{6(6-1)}{\lfloor 2}\Delta^2 + \frac{6(6-1)(6-2)}{\lfloor 3}\Delta^3 + \cdots$

$= y_0 + 6\Delta y_0 + \frac{6\times 5}{2}\Delta^2 y_0 + \frac{6\times 5\times 4}{3\times 2}\Delta^3 y_0 + \cdots$

$= 9 + 6\times 9 + 15\times(-7) + 20\times 9 + 0 + \cdots$

$= 9 + 54 - 105 + 180$

$= 243 - 105$

$= 138$ Ans