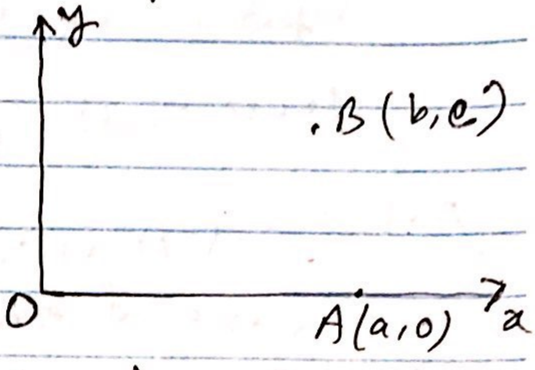
PDF-3

**Problem (1)** If the algebraic sum of the moments of a system of coplanar forces is the same and of the same sign, when taken about each of three non-collinear points, Prove that the system reduces to a couple.

**Ans**




Let O, A and B are three non-collinear points, where O is the origin.
Let the Coordinates of the points A are $(a,0)$, B are $(b,c)$ and O are $(0,0)$.
Let G be the moment of the Couple at O.

[Note: Moment of the Couple at $(h,k)$ is
$G - hY + KX$ ]

Then the moments of Couples about A $(a,0)$ is
$$G - aY + 0.X = G - aY$$

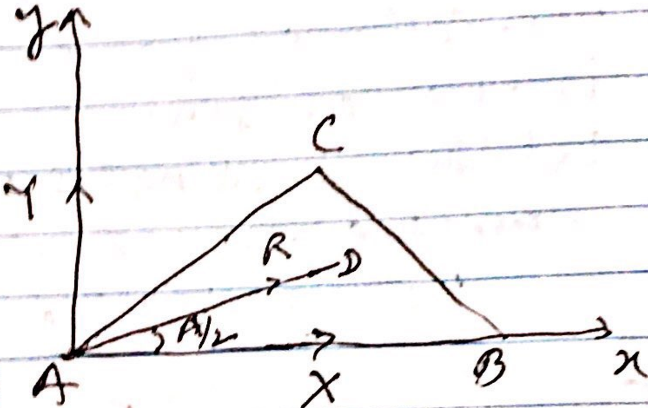
and the moment of Couples about B $(b,c)$ is
$$G - bY + c.X$$

ATB, $G = G - aY = G - bY + cX$.
Taking 1st and 2nd, we have
$G = G - aY \Rightarrow Y = 0$, since $a \neq 0$
Taking 1st and 3rd, we have
$G = G - bY + cX$
$\Rightarrow bY = cX \Rightarrow X = 0$, since $Y = 0$ and $G \neq 0$.
Hence the system of forces reduces to a couple of moment G.

Problem (2) A system of forces in the plane of a triangle ABC is equivalent to a single force at A, acting along the internal bisector of the angle BAC, and a couple of moment $G_1$. If the moments of the system about B and C are $G_2$ and $G_3$.

Prove that $(b+c)G_1 = bG_2 + cG_3$.

Ans : Let A is taken as origin and AB is the x-axis.




Let X and Y are the resolved parts of the force R along x and y axes respectively.

i.e. X, Y and $G_1$ be the system at A.

$\because \angle DAB = \frac{A}{2}$.

R is act on AD, which is bisector of $\angle BAC$.

$$\therefore \tan\frac{A}{2} = \frac{Y}{X}$$

$$\Rightarrow \frac{\sin A/2}{\cos\frac{A}{2}} = \frac{Y}{X}$$

$$\Rightarrow \frac{2\sin A/2 \cos A/2}{2\cos^2 A/2} = \frac{Y}{X}$$

$$\Rightarrow \frac{\sin A}{1+\cos A} = \frac{Y}{X}$$

$\Rightarrow (1+\cos A)Y = \sin A \cdot X$

$\Rightarrow (1+\cos A)Y - X\sin A = 0$ ——— (1)

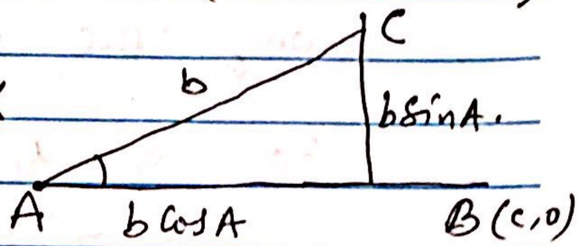
Let the Coordinates of B are $(c, 0)$.

Then $G_2 = G_1 - cY + 0 \cdot X$

$\Rightarrow G_2 = G_1 - cY$ ——— (2)

The Coordinates of C are $(b\cos A, b\sin A)$

Then $G_3 = G_1 - b\cos A\, Y + b\sin A \cdot X$ ——— (3)




Eq (2) $\times b$ + Eq (3) $\times c \Rightarrow$

$bG_2 + cG_3 = (b+c)G_1 - bc\left[(1+\cos A)Y - X\sin A\right]$

$= (b+c)G_1 - bc \times 0$ by (1)

$= (b+c)G_1$

$\therefore (b+c)G_1 = bG_2 + cG_3$ Proved.

Problem (3) The algebraic sum of the moments of a system of forces about 4 points whose Coordinates are $(x_r, y_r)$; $r = 1,2,3,4$. referred to OX and OY as axes are $G_r$; $r = 1,2,3,4$ respectively. Prove that

$$\begin{vmatrix} 1 & x_1 & y_1 & G_1 \\ 1 & x_2 & y_2 & G_2 \\ 1 & x_3 & y_3 & G_3 \\ 1 & x_4 & y_4 & G_4 \end{vmatrix} = 0$$

Solution: Let $G$ be the moment of the system about O, which is origin.

Then
$$G - x_1 Y + y_1 X = G_1$$
$$G - x_2 Y + y_2 X = G_2$$
$$G - x_3 Y + y_3 X = G_3$$
$$G - x_4 Y + y_4 X = G_4.$$

Where X and Y are resolved parts of R along the axes.

i.e
$$G - x_1 Y + y_1 X - G_1 = 0 \quad \text{——} (1)$$
$$G - x_2 Y + y_2 X - G_2 = 0 \quad \text{——} (2)$$
$$G - x_3 Y + y_3 X - G_3 = 0 \quad \text{——} (3)$$
$$G - x_4 Y + y_4 X - G_4 = 0 \quad \text{——} (4)$$

Eliminating G, Y and X from equations (1) to (4), we have

$$\begin{vmatrix} 1 & -x_1 & y_1 & -G_1 \\ 1 & -x_2 & y_2 & -G_2 \\ 1 & -x_3 & y_3 & -G_3 \\ 1 & -x_4 & y_4 & -G_4 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & x_1 & y_1 & G_1 \\ 1 & x_2 & y_2 & G_2 \\ 1 & x_3 & y_3 & G_3 \\ 1 & x_4 & y_4 & G_4 \end{vmatrix} = 0$$

Proved.