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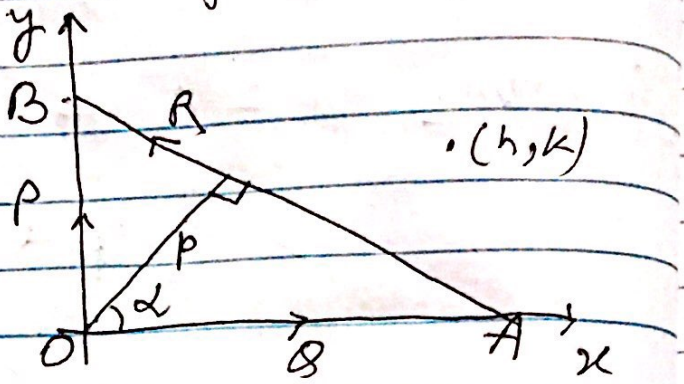
Problem (5) Forces P, Q, R act along the lines $x=0$, $y=0$ and $x\cos\alpha + y\sin\alpha = p$, Find the magnitude of the resultant and the equation of its line of action.

Solution



Let AB is a st. line intersects x and y axes at the points A and B. Its equation is

$x\cos\alpha + y\sin\alpha = p$, where p is the length of perpendicular drawn from origin upon the line AB and this $\perp^r$ makes an angle α with x-axis.

The forces P, Q and R act along the line OB, OA and AB respectively.

Then the resolved parts of the forces X and Y along ox and oy are

$$X = Q + R\cos(180^\circ - A)$$
$$= Q - R\cos A$$
$$= Q - R\cos(90^\circ - \alpha)$$
$$= Q - R\sin\alpha$$

$$\left[\because \cos(180^\circ - A) = \frac{Ox}{AB},\ \cos(180^\circ - A) = \frac{Ox}{R}\right]$$

and $Y = P + R\sin(180^\circ - A)$

$$= P + R\sin A$$
$$= P + R\sin(90^\circ - \alpha)$$
$$= P + R\cos\alpha.$$

$$\left[\sin(180^\circ - A) = \frac{Oy}{AB},\ \sin(180^\circ - A) = \frac{Oy}{R}\right]$$

$\because \angle A = 180^\circ - 90^\circ - \alpha$

Then the magnitude of the resultant of the resolved parts of the forces $= \sqrt{X^2 + Y^2}$

Now $\sqrt{X^2+Y^2} = \sqrt{(Q-R\sin\alpha)^2+(P+R\cos\alpha)^2}$

$= \sqrt{Q^2+R^2\sin^2\alpha - 2QR\sin\alpha + P^2 + R^2\cos^2\alpha + 2PR\cos\alpha}$

$= \sqrt{P^2+Q^2+R^2-2QR\sin\alpha+2PR\cos\alpha}$

Taking moments about O,

$-P\times O + Q\times O + R\times p = G$

$\therefore G = Rp$

The equation of the line of action of the resultant is $xY - yX = G$.

ie $x(P+R\cos\alpha) - y(Q-R\sin\alpha) = Rp$

This the required Equation of line of action.