9) find the first and 2nd difference of $x^4 - 6x^3 + 11x^2 - 5x + 8$ with $h=1$, show that the fourth difference is constant.

Ans:

let y be the function.

given $y = x^4 - 6x^3 + 11x^2 - 5x + 8$

let $x^4 - 6x^3 + 11x^2 - 5x + 8 = x^{(4)} + A x^{(3)} + B x^{(2)}$
$$+ C x^{(1)} + 8 \quad\text{——①}$$
$$= x(x-1)(x-2)(x-3) + A x (x-1) (x-2) + B x (x-1) + C x + 8$$

Putting $x = 1$
$$1 - 6 + 11 - 5 + 8 = C + 8$$
$$\therefore \boxed{C = 1}$$

Putting $x = 2$
$$16 - 6 \times 8 + 11 \times 4 - 10 + 8 = 2B + 2C + 8$$
$$\Rightarrow 2 = 2B + 2$$
$$\therefore \boxed{B = \frac{1}{2}}$$

Putting $x = 3$
$$81 - 6 \times 27 + 11 \times 9 = 5 \times 3 + 8$$
$$= 6A + 6B + 3C + 8$$

$$\Rightarrow 81 - 162 + 99 - 15 = 6A + 6B + 3C$$
$$\Rightarrow 180 - 177 = 6A + 6 \times \frac{1}{2} + 3 \times 1$$
$$\Rightarrow 3 - 6 = 6A$$
$$\therefore \boxed{A = -\frac{1}{2}}$$