$\therefore$ ① becomes

$$y = x^{(4)} - \frac{1}{2}x^{(3)} + \frac{1}{2}x^{(2)} + x^{(1)} + 8$$

$$\therefore \Delta y = 4x^{(3)} - \frac{1}{2}3x^{(2)} + \frac{1}{2}$$

$$\Delta^2 y = 12x^{(2)} - 3$$

$$\Delta^3 y = 24x^{(1)}$$

$$\Delta^4 y = 24$$

proved

10) Obtain the function whose first difference is $2x^3 + 3x^2 - 5x + 4$.

Ans let y be the function

given $\Delta y = 2x^3 + 3x^2 - 5x + 4$

let $2x^3 + 3x^2 - 5x + 4 = 2x^{(3)} + Ax^{(2)} + Bx^{(1)} + 4$ —— ①

$= 2x(x-1)(x-2) + Ax(x-1) + Bx + 4$

Putting $x = 1$

$2 + 3 - 5 + 4 = B + 4$

$\therefore \boxed{B = 0}$

Putting $x = 2$

$2 \times 8 + 3 \times 4 - 10 \times 4 = 2A + 2B + 4$

$\Rightarrow 18 = 2A$

$\therefore \boxed{A = 9}$