

let y be the function

given $y = 3x^4 - 4x^3 + 6x^2 + 2x + 1$

let $3x^4 - 4x^3 + 6x^2 + 2x + 1 = 3x^{(4)} + Ax^{(3)} + Bx^{(2)} + Cx^{(1)} + D \quad \text{--- (1)}$

$$\Rightarrow 3x^4 - 4x^3 + 6x^2 + 2x + 1 = 3x(x-1)(x-2)(x-3) + Ax(x-1)(x-2) + Bx(x-1) + Cx + D$$

Putting $x=0$

$$\therefore \boxed{D=1}$$

Putting $x=1$

$$3 - 4 + 6 + 2 + 1 = C + D$$

$$\therefore \boxed{C=7}$$

Putting $x=2$

$$48 - 32 + 24 + 4 + 1 = 2B + 2C + D$$

$$\Rightarrow 16 + 28 = 2B + 14$$

$$\therefore \boxed{B=15}$$

Putting $x=3$

$$243 - 108 + 54 + 6 + 1 = 6A + 6B + 3C + D$$

$$195 = 6A + 90 + 21$$

$$\therefore \boxed{A=14}$$

\therefore (1) becomes

$$y = 3x^{(4)} + 14x^{(3)} + 15x^{(2)} + 7x^{(1)} + 1$$

$$\Delta y = 12x^{(3)} + 42x^{(2)} + 30x^{(1)} + 7$$

$$\Delta^2 y = 36x^{(2)} + 84x^{(1)} + 30$$

$$\Delta^3 y = 72x^{(1)} + 84$$

$$\Delta^4 y = 72$$

$$\Delta^5 y = 0$$

