

let  $x^3 - 2x^2 + x - 1 = x^{(2)} + Ax^{(2)} + Bx^{(1)} + C$   
 $\Rightarrow x^3 - 2x^2 + x - 1 = x(x-1)(x-2) + Ax(x-1) + Bx + C$

Putting  $x=0$   
 $\therefore \boxed{-1=C}$

Putting  $x=1$   
 $1-2+1-1 = B+C$   
 $\therefore \boxed{B=0}$

Putting  $x=2$   
 $8-8+2-1 = 2A+C$   
 $\therefore \boxed{A=1}$

$\therefore$  ① becomes

$$y = x^{(3)} + x^{(2)} - 1$$

$$\Delta y = 3x^{(2)} + 2x^{(1)}$$

$$\Delta^2 y = 6x^{(1)} + 2$$

$$\Delta^3 y = 6$$

$$\Delta^4 y = 0$$

proved

7) Express  $U = x^4 - 12x^3 + 24x^2 - 30x + 9$  and its successive difference in factorial notation. Hence show that  $\Delta^5 U = 0$ .

Ans  $\therefore U = x^4 - 12x^3 + 24x^2 - 30x + 9$   
 let  $U = x^{(4)} + Ax^{(3)} + Bx^{(2)} + Cx^{(1)} + D$  — ①

$$\Rightarrow x^4 - 12x^3 + 24x^2 - 30x + 9 = x(x-1)(x-2)(x-3) + Ax(x-1)(x-2) + Bx(x-1) + Cx + D$$