Dr. P. C. Banerjee

Analytical Dynamics, P.G. - CC6

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NAME Dr. P. C. Banerjee STD. SEC. ROLL NO.

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Analytical Dynamics (P.G.-Sem-2)

Syllabus:-

Motion in two Dimension :-

A1 : Motion of C.G. and motion about C.G., K.E. Slipping of ~~rod~~ rods, motion of sphere on inclined plane when rolling and sliding are combined, motion of circular disk on a plane and related problems.

A2 : Moving axes:
velocity and accn. in two dimension motion when the axes are moving, velocity and accn. in three dimension when the axes are moving, velocity and accn. motion in polar form, angular velocity referred to moving axes and Euler's geometrical equation.

A3 : Equation of motion and its application in 3-Dim.

General equation of motion, Euler's equation of motion, momentum of rigid body, moments about instantaneous axes, K.E. of rigid body and related problems.

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Ans: Lagrange's equation of motion of Small Oscillation:

Generalized Co-ordinates, Constraints classification of mechanical system, Lagrange's equation of motion, principle of energy, Small oscillation, normal Co-ordinates.

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Books

1. Dynamics of a rigid body → Pragati Prakashan (Publ)
   Dr. B.B. Lal, Prof. R.K. Saini, Dr. Pratima Tripathi, Dr. Praveen Kumar

2. Dynamics of rigid body → Kedar Nath Ram Nath, Delhi (Publ)
   Brahma Nand, B.S. Tyagi, B.D. Sharma.

Some Important terms in Dynamics :- (Remember)

① **Rigid body** :- A rigid body is the system of particles and it does not change in shape and size in any way. The distance between any two particles of the rigid body remains always the same.

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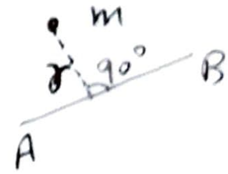
② **Moment of Inertia of a particle** :-

Let us consider a particle of mass 'm' and a line (axis) AB, then the moment of inertia (M.I.) of particle about the line (axis) $= I = mr^2$, where r = perp. distance of the particle from line AB



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③ **Moment of Inertia of a rigid body (system of particles)**:

Let the rigid body consist of particles of masses $m_1, m_2, m_3, \ldots, m_n$ and let $r_1, r_2, r_3, \ldots, r_n$ be the ⊥r. distances of these particles from a line (axis) AB, then

$$\text{M.I. of the body} = m_1 r_1^2 + m_2 r_2^2 + \cdots + m_n r_n^2$$

$$\text{i.e. } I = \sum_{i=1}^{n} m_i r_i^2$$

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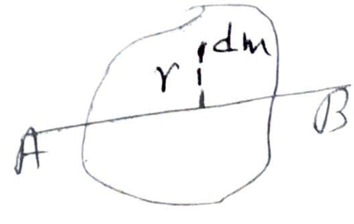
④ **Moment of Inertia of a Continuous distribution of mass**;

Let dm be elementary mass of an elementary portion of a rigid body and r = distance of dm from axis AB.



Then $M.I. = I = \int r^2 dm$ (2)

Where integration is taken over the whole body.

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(5) Radius of Gyration :-

I = M.I. of a rigid body about the line AB

$$= \sum_{i=1}^{n} m_i r_i^2$$

let M = total mass of the body $= \sum_{i=1}^{n} m_i$

we define a quantity K such that

$$I = MK^2$$

$$\Rightarrow K^2 = \frac{I}{M} = \frac{\sum_{i=1}^{n} m_i r_i^2}{\sum_{i=1}^{n} m_i}$$

$$\text{or } K^2 = \frac{\int r^2 dm}{\int dm}$$

Then K is called radius of gyration of the body about AB.

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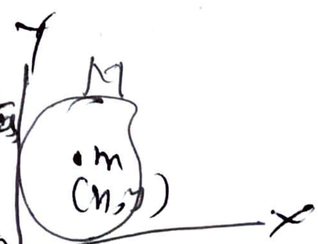
(6) Product of Inertia :-



let m = mass of a particle of co-ordinates (x,y) in two dimension

Then

(i) product of inertia of the body about OX and OY axes $= \sum mxy$

(ii) let m = mass of a particle of co-ordinates (x,y,z) in 3-dimension

~~Let~~ Then

A = M.I. ~~(moment)~~ about the axis $X = \sum m(y^2+z^2)$

B = M.I. " " " $Y = \sum m(z^2+x^2)$

C = M.I. " " " $Z = \sum m(x^2+y^2)$

A_1 = M.I. with regard to plane $YZ = \sum m x^2$

B_1 = " " " " " $ZX = \sum m y^2$

C_1 = " " " " " $XY = \sum m z^2$

H = M.I. with respect to the origin

$= \sum m(x^2+y^2+z^2) = \sum m r^2$.

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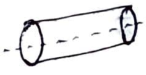
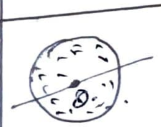
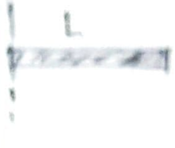
M.I. of Regular Shaped bodies :— M = Mass, K = Radius of gyration

Sl. No.	Body	Axis (line)	Figure	$I = MK^2$	K
1.	Ring of radius R	⊥r. to plane at the centre		MR^2	R
2.	"	Diameter		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$
3.	Disc, radius R	⊥r. to the plane at the centre		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$
4.	"	Diameter		$\frac{1}{4}MR^2$	$R/2$
5.	Hollow cylinder, Radius R	Axis of cylinder		MR^2	R
6.	Solid cylinder Radius R	"		$\frac{1}{2}MR^2$	$R/\sqrt{2}$
7.	Solid Sphere Radius R	Diameter		$\frac{2}{5}MR^2$	$\sqrt{\frac{2}{5}}R$
8.	Spherical Shell, Radius R	"		$\frac{2}{3}MR^2$	$\sqrt{\frac{2}{3}}R$

Sl No.	Body	Axis	Figure	I =	K =
9	Thin Rod, Length L	⊥r to rod at mid-point		$\frac{ML^2}{12}$	$\frac{L}{\sqrt{12}}$
10	"	⊥r to rod at one end		$\frac{ML^2}{3}$	$\frac{L}{\sqrt{3}}$

7) Rotational K.E.:-

$$E_K = \sum_{i=1}^{n} \frac{1}{2} m_i v_i^2 \text{ , where } v_i = r_i \omega$$

$$\therefore E_K = \sum_{i=1}^{n} \frac{1}{2} m_i r_i^2 \omega^2 = \frac{1}{2}\left(\sum_{i=1}^{n} m_i r_i^2\right)\omega^2$$

$$= \frac{1}{2} I \omega^2$$

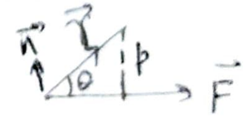
8) Work done by a force:-

$$W = \text{work done} = \vec{F} \cdot \vec{d}$$

9) Angular velocity:

$$\vec{V} = \vec{\omega} \times \vec{r}$$

Note: (10)



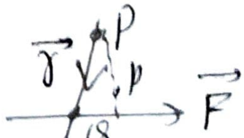
$$\vec{r} \times \vec{F} = |\vec{r}||\vec{F}| \sin\theta \, \vec{n}$$
$$= F (r \sin\theta) \vec{n}$$
$$= F p \vec{n}$$

where $p = r\sin\theta$ = Length of ⊥r in direction of $\vec{F}$

10) Moment of a force about a point:-

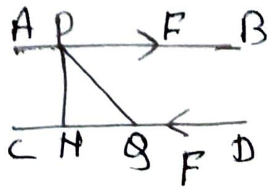
$$\vec{M} = \vec{r} \times \vec{F} = pF\vec{n}$$

where p = Length of ⊥r from pt. on Force



11) Couple:- Two equal and unlike forces are said to be consitute a couple.

The moment of the couple about any point $= F \cdot PN$, PN = Dist. betw two unlike forces



(13) <u>Impressed forces</u>: The external forces acting on a body are called impressed forces.

<u>Effective forces</u>: The internal forces inside the body are called effective forces.

If m be the mass of a particle of the body and its accele. f. Then (i) $m\frac{d^2x}{dt^2}$, $m\frac{d^2y}{dt^2}$, $m\frac{d^2z}{dt^2}$ are the effective forces on the particle parallel to the axes.

(ii) $m\frac{d^2s}{dt^2}$, $m\frac{v^2}{\rho}$ are the effective forces on the particle along the tangent and normal

(iii) $\left(-m\frac{d^2x}{dt^2}\right)$ etc. are called <u>reversed effective forces</u>

<u>D'Alembert Principle</u>:- The reversed effective forces at each point of the system and the impressed forces on the system are in equilibrium.

<u>Angular Momentum of a System of particles</u>:

Let r = position vector of a particle of mass m, relative to point O then the vector sum

$$H = \sum r \times mv = \sum m r \times v$$

is called <u>angular momentum</u> (or <u>moment of momentum</u>) of the system about O.

<u>Centroid of a System</u>:

Let r = position vector of a particle of mass m relative to the point O, then the point with position vector

$$\bar{r} = \frac{\sum m r}{\sum m}$$ is the centroid of the system.

If v = velocity of the particle of mass m

V = " vector of the centroid, then

$$V = \frac{d\bar{r}}{dt} = \frac{\sum m \frac{dr}{dt}}{\sum m} = \frac{\sum mv}{\sum m}$$

General Equations of a Motion for D'Alembert principle

(i) Vector form:

$\vec{r}$ = position vector of a particle of mass m of the system in time t.

$\vec{F}$ = external force acting on it, then

$$\boxed{\begin{aligned} \sum m \frac{d^2\vec{r}}{dt^2} &= \sum \vec{F} \\ \text{and} \quad \sum m\vec{r} \times \frac{d^2\vec{r}}{dt^2} &= \sum \vec{r} \times \vec{F} \end{aligned}}$$

These two are general vector equations of motion of a rigid body.

(ii) Cartesian form:

Let $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k} = (x, y, z)$

the components of $\vec{F}$ are X, Y, Z

Then

$$\boxed{\begin{aligned} \sum m \frac{d^2x}{dt^2} &= \sum X \\ \sum m \frac{d^2y}{dt^2} &= \sum Y \\ \sum m \frac{d^2z}{dt^2} &= \sum Z \\ \sum m\left(y \frac{d^2z}{dt^2} - z \frac{d^2y}{dt^2}\right) &= \sum (yZ - zY) \\ \sum m\left(z \frac{d^2x}{dt^2} - x \frac{d^2z}{dt^2}\right) &= \sum (zX - xZ) \\ \sum m\left(x \frac{d^2y}{dt^2} - y \frac{d^2x}{dt^2}\right) &= \sum (xY - yX) \end{aligned}}$$

These six are the scalar general equations of motion of any rigid body.

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M.Sc.

Motion in Two Dimension

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(1) Art: Determine the dynamical equation of motion in two dimension when the acting forces are finite.

Ans:- We know that the motion of rigid body is a combination of two independent motions i.e.,

(i) The motion of a centre of inertia, and

(ii) The motion about the centre of inertia.

Here (i) states that the motion of centre of intertia is such that the total mass M of the rigid body concentrate at the C.G. and all the external forces are transferred parallel to themselves to act at the C.G.

Let $\vec{r}$ = position vector of the C.G.

and $\vec{F}$ = external force acting at any particle of mass 'm' of the body,

then

$$M\frac{d^2\vec{r}}{dt^2} = \sum\vec{F} \longrightarrow (1)$$

Let $(\bar{x}, \bar{y})$ = Co-ordinates of C.G.

and X, Y = components of the force F parallel to the axes, then

$$\vec{r} = \bar{x}\vec{i} + \bar{y}\vec{j} \quad \text{and} \quad \vec{F} = X\vec{i} + Y\vec{j}$$

Hence (1) becomes