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Thm: A subset F of a metric space X is closed if & only if $F = \bar{F}$ i.e. $F = cl(F)$.

Proof Let F is a closed subset of a metric space X.

$\Rightarrow F' \subseteq F$

$\Rightarrow F \cup F' = F$

$\Rightarrow \bar{F} = F$ (Proved)

Conversely Let $F = \bar{F}$

Let x be any limit point of F

$\Rightarrow x \in F'$

$\Rightarrow x \in F \cup F' = \bar{F}$

$\Rightarrow x \in F$ ($\because F = \bar{F}$)

Hence F is closed.

Thm Let F be any subset of metric space X. Then prove that $\bar{F}$ is a closed superset of F which is contained in every closed superset of F.

Proof Since $\bar{F} = F \cup F'$

$\Rightarrow \bar{F}$ is super set of F.

Now let x is any limit point of $\bar{F}$. Then the open sphere $S_r(x)$ contains one point y of $\bar{F}$ different from x. Now two cases arise

(i) either $y \in F$ or (ii) $y \in F'$

Case I If $y \in F$, mean every open sphere centred at x contains a point of F other than x. So x is limit point of F. i.e. $x \in F'$

$$\therefore \quad x \in F \cup F' = \overline{F}$$

Case II If $y \in F'$ then every open sphere centred at x contains a point of F' other than x. So x is limit point of F'. We know that F' is closed set so it will contain x, i.e. $x \in F'$

$$\therefore \quad x \in F \cup F' = \overline{F}$$

Hence $\overline{F}$ is closed set.

Now let G be any other closed set containing F.
We have to show that $\overline{F} \subset G$

Let $x \in \overline{F}$ i.e. $x \in F \cup F'$

$\Rightarrow$ Either $x \in F$ or $x \in F'$

If $x \in F$ then $x \in G$ ($\because F \subseteq G$) —— (1)

If $x \in F'$ then x is limit point of F

$\Rightarrow$ For every $r > 0$, $S_r(x)$ contains at least one point y of F other than x

$\Rightarrow$ $S_r(x)$ contains at least one point y of G other than x
($\because F \subseteq G$)

$\Rightarrow$ x is limit point of G —— (2)

$\therefore$ From (1) & (2) we

$\Rightarrow$ $\overline{F} \subseteq G$, i.e. $\overline{F}$ is smallest closed set containing F. (Proved)

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Convergence:

Let (X,d) be a metric space. Let $\{x_n\}$ be sequence of points in X. A point $x \in X$ is said to be limit of sequence $\{x_n\}$ if

$$\lim_{n\to\infty} d(x_n, x) = 0$$

ie For every $\epsilon > 0$, $\exists$ an integer $m > 0$ s.t.

$$x_n \in S_\epsilon(x) \quad \forall \quad n \geq m.$$

or

$$d(x_n, x) < \epsilon \quad \forall \quad n \geq m.$$

Equivalent matrics:

Let X be a non-empty set and d_1 & d_2 be two metrics on X, then d_1 is said to stronger than d_2 if for any sequence $\{x_n\}$ in X

$$x_n \xrightarrow{d_1} x \text{ implies } x_n \xrightarrow{d_2} x$$

Two metrics on a non-empty set X is said to be equivalent if d_1 is stronger than d_2 & d_2 stronger than d_1

Cauchy Sequence:

A sequence $\{x_n\}$ of points in a metric space (X,d) is said to be Cauchy sequence if for every $\epsilon > 0$ $\exists$ an integer $n_\epsilon > 0$ s.t.

$$d(x_m, x_n) < \epsilon \quad \forall \quad n > n_\epsilon$$

ie $\quad d(x_m, x_n) \to 0$ as $m, n \to \infty$

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