

Probable questions for

P.G. - Sem-II

Analytical Dynamics, CC-206

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1. Determine the dynamical equation of motion in two dimension when the acting forces are finite.
2. Find The moment of momentum (or angular momentum) of a rigid body about the fixed origin  $O$ , when the body is moving in two dimension.
3. Find the K.E. of a body moving in 2-dimension.
4. A uniform sphere rolls down an inclined plane, rough enough to prevent any sliding. Discuss the motion.
5. A cylinder rolls down a smooth plane. whose inclination to the horizontal is  $\alpha$ , untrapped, as it goes, a fine string fixed to the highest point of the plane. Find its acceleration and the tension of the string.
6. A uniform solid cylinder is placed with its axis horizontal on a plane, whose inclination

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to the horizon is  $\alpha$ , show that the least co-efficient of friction between it and the plane, so that it may roll and not slide, is  $\frac{1}{3} \tan \alpha$ . If the cylinder be hollow, and of small thickness, the least value is  $\frac{1}{2} \tan \alpha$ .

7. A uniform rod is held in a vertical position with one end resting upon a perfectly rough table and when released rotates about the end in contact with the table. Discuss the motion.

8. A uniform rod is held at an inclination  $\alpha$  to the horizon with one end in contact with a horizontal table whose co-efficient of friction is  $\mu$ . If it be released, show that it will commence to slide if

$$\mu < \frac{3 \sin \alpha \cos \alpha}{1 + 3 \sin^2 \alpha}$$

9. A rough uniform rod of length  $2a$ , is placed on a rough table at right to its edge, if its centre of gravity be initially at distance  $b$  beyond the edge, show that the rod will begin to slide when it has turned through an angle  $\tan^{-1} \frac{a^2}{a^2 + 9b^2}$ .

10. A sphere of radius 'a' whose centre of gravity  $G_c$  is at a distance  $c$  from its Centre  $C$  is placed on a rough plane so that  $C.G_c$  is horizontal. Show that it will begin to roll or slide according as the co-efficient of friction  $\mu >$  or  $< \frac{ac}{K^2+a^2}$ , where  $K$  is the radius of gyration about a horizontal axis through  $G_c$ .
11. Obtain Lagrange's equation of motion for holonomic conservative system of forces.
12. Obtain the equation giving K.E. in terms of generalised co-ordinates which does not contain time  $t$ .
13. If Lagrangian function does not contain time explicitly, prove that the total energy of the conservative system is conserved.
14. A particle of mass  $M$  moves in a conservative force field. Find (a) the Lagrangian and (b) the equation of motion in cylindrical form.
15. Set up Lagrangian equation of a particle moving on the surface of earth using spherical polar co-ordinates.

16. A particle of mass  $M$  is moving in a plane and attracted towards the origin of the co-ordinates with a force inversely proportional to the square of the distance from it. Set up the Lagrangian and hence obtain the equation describing its motion.
17. A uniform rod, of mass  $3m$  and length  $2l$ , has its middle point fixed and a mass  $m$  attached at one extremity. The rod when in a horizontal position is set rotating about a vertical axis through its centre with an angular velocity equal to  $\sqrt{\frac{2ng}{l}}$ . Show that the heavy end of the rod will fall till the inclination of the rod to the vertical is  $\cos^{-1}(\sqrt{n^2+1}-n)$  and will then rise again.
18. Set up the Lagrangian for a simple pendulum and obtain an equation describing its motion.
19. Use Lagrange's equation to find the differential equation for a compound pendulum, which oscillates in a vertical plane about a fixed horizontal axis.

20. A uniform rod of length  $2a$ , which has one end attached to a fixed point by a light inextensible string of length  $\frac{5}{12}a$ , performing small oscillation in a vertical plane about its position of equilibrium. Find the position at any time and show that the period of its principal oscillations are  $2\pi\sqrt{\frac{5a}{3g}}$  and  $\pi\sqrt{\frac{a}{3g}}$ .
21. A uniform str. rod of length  $2a$  is freely movable about its centre and a particle of mass one third that of the rod is attached by a light inextensible string of length  $a$ , to one end of the rod. Show that the one period of the principal oscillation is  $(\sqrt{5} + 1)\pi\sqrt{\frac{a}{g}}$ .
22. A uniform bar of length  $2a$  is hung from a fixed point by a string of length  $b$  fastened to one end of the bar. Show that when the system makes small normal oscillations in a vertical plane, the length  $l$  of the equivalent simple pendulum is a root of the quadratic equation  $l^2 - (\frac{4a}{3} + b)l + \frac{ab}{3} = 0$ .
23. Find the velocity of a point when a rigid body is turning about a fixed point  $O$ .

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24. Find K.E. of moving body in 3-dimension.
25. Derive Euler's dynamical equations of motion about a fixed point of a rigid body.
26. Describe Eulerian angles
27. Obtain the integrals of energy and angular momentum when a rigid body rotates about a fixed point under no external forces.
28. Deduce Euler's equations from Lagrange's equations.
29. What do you mean by Invariable line? And describe it.
30. A body under the action of no external forces turns so that resolved part of its angular velocity about one of the principal axes at centre of gravity is constant. Show that the angular velocity of the body must be constant.

Books: Dynamics of a rigid body  
Pragati Prakashan  
Dr. P. Tripathi, Dr. P. Kumar

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