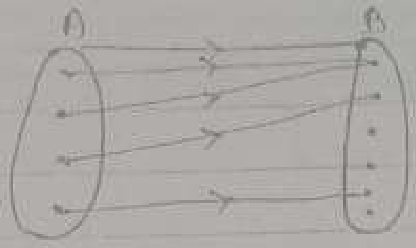
MAPPING (function) (In Short for next ch.)

The relation of subset of $A \times B$ is called a mapping if (i) Each element of A must associated with some element of B

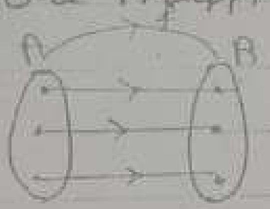
(ii) Each element of A have unique image in B.

We write $f: A \to B$

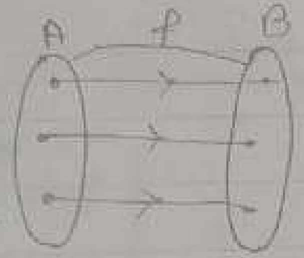


Hence A is domain, B is co-domain.

One-one mapping –

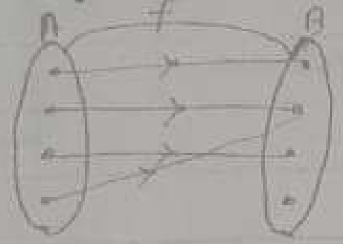


one-one onto maping.

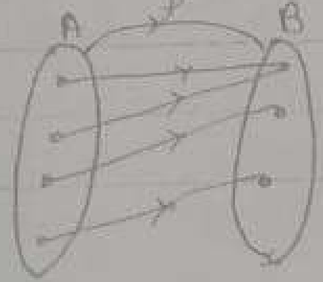


one-one onto mapping.

Many-one mapping –



Many one into mapping



Many-one onto mapping

$\ni$: (such that)

Range - The set of images is called range and it is the subset of co-domain is B

One-One mapping -

(i) $f(x) = f(y) \Leftrightarrow x = y$

(ii) $x \neq y \Rightarrow f(x) \neq f(y)$

Problems :-

① If I be the set of integers and a binary operation ⊕ defined as $a \oplus b = a+b+1$ for $a, b \in I$ then prove that I is an abelian group.

Ans → We verify the following group properties :

(i) Closure property -

If $a, b \in I$ then $a+b+1 \in I$. Since $a+b+1$ is an unique integer $a \oplus b \in I$

Hence closure property hold in I

(ii) Associative property :-

Let $a, b, c \in I$

Then $a \oplus (b \oplus c) = a \oplus (b+c+1)$

$= a + (b+c+1) + 1$

$= a+b+c+2$

and,

$(a \oplus b) \oplus c = (a+b+1) \oplus c$

$= a+b+c+2$

$\therefore a \oplus (b \oplus c) = (a \oplus b) \oplus c$

i.e Associative property hold in I

(iii) Existence of identity → Let e be the identity ~~element~~ element.

$\therefore a \oplus e = a$

$\Rightarrow a + e + 1 = a$

$\Rightarrow e = -1 \in I$

We have,

$a \oplus (-1) = a + (-1) + 1 = a$

and,

$(-1) \oplus a = -1 + a + 1 = a$

$-1 \in I$ and which is the identity element.

Hence identity element exists.

(iv) Existence of inverse → Let a^{-1} be inverse of a

$\therefore a + a^{-1} = -1$

$a + a^{-1} + 1 = -1$

$a^{-1} = -2 - a \in I$

Let $a \in I$ since $a \oplus (-a-2) = -1$

So, $-a-2$ is inverse of a

Hence inverse of each element exists. Hence I is a group under operation $\oplus$. Also I is abelian

Since,

$a \oplus b = a + b + 1$

$= b + a + 1 = b \oplus a \quad \forall a, b \in I$

(2) If R be the set of real numbers and a binary operation $*$ defined as $a * b = \frac{1}{2}ab$ is this form on abelian group

Ans- We verify the following group axioms as the set of real numbers.

(i) Closure property-

$\because a * b = \frac{1}{2}ab \in R$

as, $a, b \in R$

$\therefore$ closure property hold in R

(ii) Associative property-

Let $a, b, c \in R$

Then, $a*(b*c) = a*(\frac{1}{2}bc) = \frac{1}{2}a(\frac{1}{2}bc) = \frac{1}{4}abc$

and $(a*b)*c = (\frac{1}{2}ab)*c$

$= \frac{1}{2}(\frac{1}{2}ab)c = \frac{1}{4}abc$

$\therefore a*(b*c) = (a*b)*c$

i.e Associative property hold in R.

(iii) Existence of identity -

Let e be the identity element.

$\therefore a*e = a \Rightarrow \frac{1}{2}ae = a$

$\Rightarrow e = 2$

$\therefore a*2 = \frac{1}{2}ae = a$

and $2*a = \frac{1}{2}2a = a$

As $2 \in R$ and which is the identity element.

Hence the identity element exists.

$a*e = a$

$\Rightarrow \frac{1}{2}ae = a \quad \Rightarrow e = 2$

(iv) Existence of inverse.

Let $a \in R$

$a * \frac{4}{a} = 2$ and $\frac{4}{a} * a = 2$

$\therefore a * a^{-1} = 2$

$\frac{1}{2}aa^{-1} = 2$

$a^{-1} = \frac{4}{a}$

so. $\frac{4}{a}$ is the inverse of 'a' and $\frac{4}{a} \in R$

Then inverse of each element exists

Then R is a group under given binary operation $*$.

It is also abelian

Since $a * b = \frac{1}{2}ab = \frac{1}{2}ba = b * a \quad \forall a, b \in R$

③ Prove that the set of all complex number of the form $|z| = 1$ where z is a complex number is an abelian group under multiplication.

Ans:- Let L be the set of all complex numbers z of the form $|z| = 1$

Let $L = \{ z ; z = \cos\theta + i\sin\theta = e^{i\theta} \}$

Let

$$z_1 = e^{i\theta_1}, \; z_2 = e^{i\theta_2}, \; z_3 = e^{i\theta_3} \in L$$

(i) Closure property-

$$z_1 \times z_2 = e^{i\theta_1} \cdot e^{i\theta_2} = e^{i(\theta_1 + \theta_2)}$$

$$\because |z_1 \times z_2| = 1 \Rightarrow z_1 \times z_2 \in L$$

i.e closure property hold in L

(ii) Associative law $\rightarrow$

We have, $z_1 \times (z_2 \times z_3) = e^{i\theta_1} \times (e^{i\theta_2} \times e^{i\theta_3})$

$$= e^{i\theta_1} e^{i(\theta_2 + \theta_3)}$$

$$= e^{i(\theta_1 + \theta_2 + \theta_3)}$$

$$= e^{i(\theta_1 + \theta_2)} e^{i\theta_3}$$

$$= (z_1 \times z_2) \times z_3$$

Thus associative law hold in L.

(iii) Existence of identity $\rightarrow$ The identity element is

$e^{i0} = 1 \in L$, since

$z_1 \times 1 = 1 \times z_1 = z_1 \quad \forall z \in L$

Hence identity element exists.

(iv) Existence of inverse -

The inverse of $z_1 = e^{i\theta_1} \in L$ is $\frac{1}{z_1} = e^{-i\theta_1} \in L$

Since,

$$z_1 \times \frac{1}{z_1} = \frac{1}{z_1} \times z_1 = 1$$

ie inverse of each element exists in L

(v) Commutative law →

We have,

$$z_1 \times z_2 = e^{i\theta_1} e^{i\theta_2} = e^{i(\theta_1+\theta_2)}$$
$$= e^{i(\theta_2+\theta_1)} = e^{i\theta_2} e^{i\theta_1} = z_2 \times z_1 \quad \forall z_1 z_2 \in L$$

Hence from (i) to (v)

it follows that L is an abelian group under multiplication

(4) If an operation $\oplus$ is defined on the set of real numbers as $a \oplus b = a - b$, is this a group !

Ans→ Let R be the set of real numbers.

$\therefore a \oplus b = a - b \in R$ as $a, b \in R$

ie closure property hold in R

Let $a, b, c \in R$

Now, $a \oplus (b \oplus c) = a \oplus (b - c)$
$= a - (b - c) = a - b + c$

and,

$(a \oplus b) \oplus c = (a - b) \oplus c$
$= a - b - c$

$a \oplus (b \oplus c) \neq (a \oplus b) \oplus c$