29/4/2020 Now we are going to solve the linear Equation when RHS $\neq 0$.

Case ① when RHS of Eq ① ie $X = e^{ax}$, where a being any constant.

Then for finding P.I.

$$y = \frac{1}{f(D)} e^{ax}$$ and the solution is

Just replacing D by a

ie P.I. $$y = \frac{1}{f(a)} e^{ax}.$$

Problem ⑤ Solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{4x}$

Ans We write the given equation as

$$(D^2 - 5D + 6)y = e^{4x}$$

A.E. is

$$m^2 - 5m + 6 = 0$$

$$\Rightarrow m^2 - 3m - 2m + 6 = 0$$

$$\Rightarrow m(m-3) - 2(m-3) = 0$$

$$\Rightarrow (m-2)(m-3) = 0$$

Either $m = 2$ or $m = 3$

C.F. is

$$y = c_1 e^{2x} + c_2 e^{3x}$$

For P.I.

$$y = \frac{1}{D^2 - 5D + 6} e^{4x}$$

$$= \frac{1}{4^2 - 5.4 + 6} e^{4x} = \frac{e^{4x}}{2}$$

Case (2) When $X = \sin ax$ or $\cos ax$

Then for finding P.I.

$$y = \frac{1}{f(D)} \sin ax \text{ or } \cos ax$$

$$\Rightarrow y = \frac{F(D)}{\phi(D^2)} \sin ax \text{ or } \cos ax$$

For P.I. replacing D^2 by $-a^2$, we have

$$y = \frac{F(D)}{\phi(-a^2)} \sin ax \text{ or } \cos ax.$$

Problem (6) Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 2\sin 3x$

Ans $(D^2 - 3D + 1)y = 2\sin 3x$

A.E. $m^2 - 3m + 1 = 0$

$$m = \frac{-(-3) \pm \sqrt{(-3)^2 - 4.1.1}}{2.1} = \frac{3 \pm \sqrt{5}}{2}$$

C.F. is

$$y = c_1 e^{\frac{3+\sqrt{5}}{2}x} + c_2 e^{\frac{3-\sqrt{5}}{2}x}$$

P.I.

$$y = \frac{2}{D^2 - 3D + 1} \sin 3x$$

$$\Rightarrow y = \frac{2}{-3^2 - 3D + 1} \sin 3x \quad (\text{Replacing } D^2 \text{ by } -3^2)$$

$$\Rightarrow y = \frac{2}{-3D - 8} \sin 3x$$

$$\Rightarrow y = -\frac{2(3D-8)}{(3D+8)(3D-8)} \sin 3x$$

$$\Rightarrow y = \frac{-2(3D-8)}{9D^2 - 64} \sin 3x$$

$$\Rightarrow y = \frac{-2(3D-8)\sin 3x}{9(-3^2) - 64}$$

$$\Rightarrow y = \frac{-2(3.3\cos 3x - 8\sin 3x)}{-81 - 64}$$

$$\Rightarrow y = \frac{2}{145}(9\cos 3x - 8\sin 3x)$$

$\therefore$ The Complete Solution is $y = C.F. + P.I.$

i.e. $y = c_1 e^{\frac{3+\sqrt{5}}{2}x} + c_2 e^{\frac{3-\sqrt{5}}{2}x} + \frac{2}{145}(9\cos 3x - 8\sin 3x)$

Ans

Case (3). When $X = x^m$, m being a positive integer.

Then for P.I.

$$y = \frac{1}{f(D)} x^m = \frac{1}{\phi(1 \pm D)} x^m$$

Convert $f(D)$ in the form $\phi(1 \pm D)$.

Now some formulae of Binomial Th. keep in mind. These formulae will be used frequently.

1. $(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - \dots \infty$
2. $(1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 + \dots \infty$
3. $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \dots \infty$
4. $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \dots \infty$
5. $(1+x)^n = 1 + nx + \frac{n(n-1)}{\lfloor 2} x^2 + \frac{n(n-1)(n-2)}{\lfloor 3} x^3 + \dots \infty$

Problem (7) Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = x^3$

Ans The given Eq. may be written as

$$(D^2 + D + 1) y = x^3$$