

Corrd. Problem 7  
30.4.2020

A.E. is  $m^2 + m + 1 = 0$

$$m = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{-1 \pm \sqrt{-3}}{2}$$

$$m = \frac{-1 \pm \sqrt{3}i}{2}$$

C.F.

$$y = e^{-\frac{1}{2}x} \left( A \cos \frac{\sqrt{3}}{2}x + B \sin \frac{\sqrt{3}}{2}x \right)$$

Part I.I.

$$y = \frac{1}{D^2 + D + 1} (x^3)$$

$$y = (1 + D + D^2)^{-1} (x^3)$$

$$y = \{1 + (D + D^2)\}^{-1} (x^3)$$

$$y = \{1 - (D + D^2) + (D + D^2)^2 - (D + D^2)^3 + \dots\} x^3$$

$$y = x^3 - (3x^2 + 3 \cdot 2x) + (D^2 + 2D^3 + D^4)x^3 - (D^3 + 3D^4 + 3D^5 + D^6)x^3 + \dots$$

$$y = x^3 - 3x^2 - 6x + 3 \cdot 2x + 2 \cdot 3 \cdot 2 \cdot 1 + 0 = 0$$

$$y = x^3 - 3x^2 + 12$$

$\therefore$  Complete solution is

$$y = e^{-\frac{x}{2}} \left( A \cos \frac{\sqrt{3}}{2}x + B \sin \frac{\sqrt{3}}{2}x \right) + x^3 - 3x^2 + 12$$

Ans

Case (4) If  $x = e^{ax} \cdot v$ , where  $v$  is any fun of  $x$ .

Then for P.I.  $y = \frac{1}{f(D)} e^{ax} \cdot v$   
 $= e^{ax} \frac{1}{f(D+ax)} \cdot v$

Problem (8) Solve  $\frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = x^3 e^{3x}$

Ans let us write the given equation as  
 $(D^2 - 5D + 6)y = x^3 e^{3x}$ , where  $D \equiv \frac{d}{dx}$

For C.F. A.E. is

$$m^2 - 5m + 6 = 0, \text{ where } D = m$$

$$\Rightarrow m^2 - 3m - 2m + 6 = 0$$

$$\Rightarrow m(m-3) - 2(m-3) = 0$$

$$\Rightarrow (m-1)(m-3) = 0$$

i.e.  $m = \text{either } 2 \text{ or } 3$

C.F.  $y = C_1 e^{2x} + C_2 e^{3x}$

For P.I.  $y = \frac{1}{D^2 - 5D + 6} x^3 e^{3x}$

$$y = e^{3x} \frac{1}{(D+3)^2 - 5(D+3) + 6} (x^3)$$

$$y = e^{3x} \frac{1}{D^2 + 6D + 9 - 5D - 15 + 6} (x^3)$$
$$= e^{3x} \frac{1}{D^2 + D - 0} (x^3)$$

$$y = \frac{e^{3x}}{D(D+1)} x^3$$

$$y = e^{3x} \cdot \left[ \frac{1}{D} - \frac{1}{D+1} \right] (x^3)$$

$$= e^{3x} \left[ \frac{1}{D} (x^3) - (1+D)^{-1} (x^3) \right]$$

$$= e^{3x} \cdot \left[ \frac{x^4}{4} - (1 - D + D^2 - D^3 + D^4 - \dots) x^3 \right]$$

$$= e^{3x} \cdot \left[ \frac{x^4}{4} - (x^3 - 3x^2 + 3 \cdot 2x - 3 \cdot 2 \cdot 1 + 0) \right]$$

$$= e^{3x} \left[ \frac{x^4}{4} - x^3 + 3x^2 - 6x + 6 \right]$$

∴ Complete solution is

$$y = C_1 e^{2x} + C_2 e^{3x} + e^{3x} \left( \frac{x^4}{4} - x^3 + 3x^2 - 6x + 6 \right)$$

Problem (9) Solve  $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 4y = e^x \cos x$

Ans Let us write the given equation as

$$(D^2 - 2D + 4)y = e^x \cos x$$

A.E.  $m^2 - 2m + 4 = 0$

$$\Rightarrow m = \frac{-(-2) \pm \sqrt{4 - 4 \cdot 1 \cdot 4}}{2 \cdot 1}$$

$$\Rightarrow m = \frac{2 \pm \sqrt{-12}}{2} = \frac{2 \pm 2\sqrt{3}i}{2} = (1 \pm \sqrt{3}i)$$

C.F.  $y = e^x (A \cos \sqrt{3}x + B \sin \sqrt{3}x)$

For P.I:  $y = \frac{1}{D^2 - 2D + 4} e^{ix} \cos x$

$$y = e^{ix} \frac{1}{(D+1)^2 - 2(D+1) + 4} \cos x$$

$$y = e^{ix} \frac{1}{D^2 + 2D + 1 - 2D - 2 + 4} \cos x$$

$$y = e^{ix} \frac{1}{D^2 - 1} (\cos x)$$

$$y = e^{ix} \frac{1}{-1^2 - 1} \cos x = \frac{e^{ix} \cos x}{-2}$$

Complete solution is

$$y = e^{ix} (A \cos \sqrt{3}x + B \sin \sqrt{3}x) - \frac{1}{2} e^{ix} \cos x$$

Case (4) If  $X = x^m \cdot v$ .

If  $v$  is the function of  $x$ . Then combine this  $v$  with  $x^m$ .

and if  $v$  is in the form of  $\cos ax$  or  $\sin ax$ .

then apply Euler's formula i.e.  $e^{iax} = \cos ax + i \sin ax$ .

if  $v = \cos ax$ , then write  $v = R.P.$  of  $e^{iax}$

if  $v = \sin ax$ , then write  $v = I.P.$  of  $e^{iax}$

then find P.I. as finding as in case (3).

Problem (10) Solve  $\frac{d^2 y}{dx^2} + a^2 y = x \cos nx$

Ans writing the given eq. as

$$(D^2 + a^2)y = x \cos nx$$

A.E.  $m^2 + a^2 = 0$

$$m^2 - i^2 a^2 = 0$$

$$\Rightarrow (m + ia)(m - ia) = 0$$

$$\Rightarrow m = \text{either } -ia \text{ or } ia$$

$$\Rightarrow m = 0 \pm ia$$

C.F.  $y = e^{0x} (A \cos ax + B \sin ax) = A \cos ax + B \sin ax$

P.I.  $y = \frac{1}{D^2 + a^2} (x \cos nx)$   
 $= \frac{1}{D^2 + a^2} x \cdot \text{R.P. of } e^{inx}$

$$= \text{R.P. of } e^{inx} \cdot \frac{1}{(D+n)^2 + a^2} (x)$$

$$= \text{R.P. of } e^{inx} \cdot \frac{1}{D^2 + 2Dn + n^2 + a^2} (x)$$

$$= \text{R.P. of } e^{inx} \cdot \frac{1}{(n^2 + a^2) \left\{ 1 + \frac{D^2 + 2Dn}{n^2 + a^2} \right\}} (x)$$

$$= \text{R.P. of } e^{inx} \cdot \frac{1}{n^2 + a^2} \left\{ 1 + \frac{D^2 + 2Dn}{n^2 + a^2} \right\}^{-1} (x)$$



$$y = \frac{1}{n^2 + a^2} \text{R.P. of } e^{inx} \left\{ 1 - \frac{D+2Dn}{n^2+a^2} + \left( \frac{D+2Dn}{n^2+a^2} \right)^2 - \dots \right\} x$$

$$y = \frac{1}{n^2 + a^2} \text{R.P. of } e^{inx} \left\{ x - \frac{0+2n \cdot 1}{n^2+a^2} + 0 \right\}$$

$$= \frac{1}{n^2 + a^2} \text{RP} (\cos nx + i \sin nx) \cdot \left( x - \frac{2n}{n^2 + a^2} \right)$$

$$= \frac{1}{n^2 + a^2} \left( x - \frac{2n}{n^2 + a^2} \right) \cos nx$$

$$= \frac{1}{(n^2 + a^2)^2} \{ (n^2 + a^2)x - 2n \} \cos nx$$

$\therefore$  The complete solution is

$$y = \text{C.F.} + \text{P.I.}$$

$$y = A \cos ax + B \sin ax + \frac{(n^2 + a^2)x - 2n}{(n^2 + a^2)^2} \cos nx$$

Santosh  
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Ans