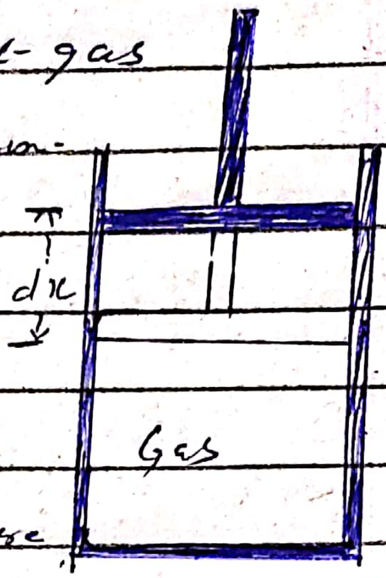
Equation of an adiabatic process:-

Let on mole of a perfect gas is contained in a cylinder of non-conducting wall and provided with a non-conducting and frictionless piston. It is shown in figure.



Let P, V and T be the pressure volume & temperature of the gas.

Let the gas be expand and piston moves through a small distance dx.

Let dU be the change in internal energy of the gas and dW be the small work done by the gas. Then, according to first Law of thermodynamics

$$dQ = dU + dW \quad \text{—(i)}$$

Since the process is adiabatic

$$\therefore \quad dQ = 0$$

Hence from eqn (i) we get-

$$dW + dU = 0 \quad \text{—(ii)}$$

If the temp of the gas decreased by dT.

Therefor the decrease in internal energy of the gas

i.e $\quad dU = C_V dT$ —(iii) $\qquad C_V$ = specific heat at ~~const volume~~

and work done

$$dW = F dx = (PA)\,dx$$ $\qquad A$ = area of cross section of the piston

$$= P(A dx)$$

$$\Rightarrow \quad dW = PdV \quad \text{—(iv)}$$ $\quad dV$ = change in volme of the gas

putting the value of eqn (iii) & (iv) in eqn (ii) we get-

$$C_v dT + p dv = 0 \quad \text{—(v)}$$

From ideal eqn we have

$$pv = RT \qquad \text{if } n=1$$

differentiating both sides of above eqn we get-

$$p dv + v dp = R dT$$

or $\cancel{dT = \frac{p dv + v dp}{R}}$

$$dT = \frac{p dv + v dp}{C_p - C_v} \quad \text{—(vi)} \qquad \because C_p - C_v = R$$

$\therefore$ eqn (v) may be written as

$$C_v\left(\frac{p dv + v dp}{C_p - C_v}\right) + p dv = 0$$

or $C_v p dv + C_v v dp + C_p p dv - C_v p dv = 0$

or $C_v v dp + C_p p dv = 0$

Dividing both sides by C_v, P, V we get-

or $\frac{C_p}{C_v}\frac{dv}{v} + \frac{dp}{p} = 0$

or $\gamma \frac{dv}{v} + \frac{dp}{p} = 0 \qquad \because \frac{C_p}{C_v} = \gamma$

~~integrating both sides~~

$$\gamma\int \frac{dv}{v} + \int \frac{dp}{p} = K \qquad K = \text{Const of integration}$$

or $\cancel{\gamma \log_e v + \log_e p = k}$ or $\cancel{\log_e v^\gamma + \log_e p = k}$

or $\log_e pv^\gamma = k$

or $pv^\gamma = e^k$

$$\Rightarrow \boxed{pv^\gamma = \text{Constant}}$$

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